

Sovereign Debt Crises and Monetary Policy

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Introduction

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 - * Nervous investors can pressure high interest rates, increasing future payments
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Q: How should a central bank in a monetary union set monetary policy to reduce vulnerability of its members?

In this paper

- Simple model of self-fulfilling debt crisis a la Calvo with nominal rigidities
Calvo (1988), Ayres et al. (2016), Lorenzoni and Werning (2013)...
- * Sticky price for NT – macro-stabilization role for fiscal and monetary policy
- * Monetary union: central bank balances heterogeneous impact of monetary policy

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- The monetary policy is time-inconsistent
 - * Central Bank(CB) has incentives to promise an expansionary monetary policy
 - * Ex-ante promises monetary easing to reduce current spreads
 - * Ex-post deviate to avoid the cost of monetary easing

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- Endogenous fragmentation → *Heterogeneity among members*
 - * Monetary unions are subject to multiplicity
 - * Safe equ: low interest rates and full employment for all members
 - * Fragmented equ: high interest rates for some members and full employment unfeasible.

Model

A Two Period Model

- Monetary Union

- * Small open monetary union

- + World interest rate exogenous \Leftarrow Small monetary union

- * Continuum of ex-ante identical countries

- * Ex-post heterogeneous in the interest rate

- * Face aggregate risk regarding default cost

- + Bimodal distribution Ayres et al. (2023)

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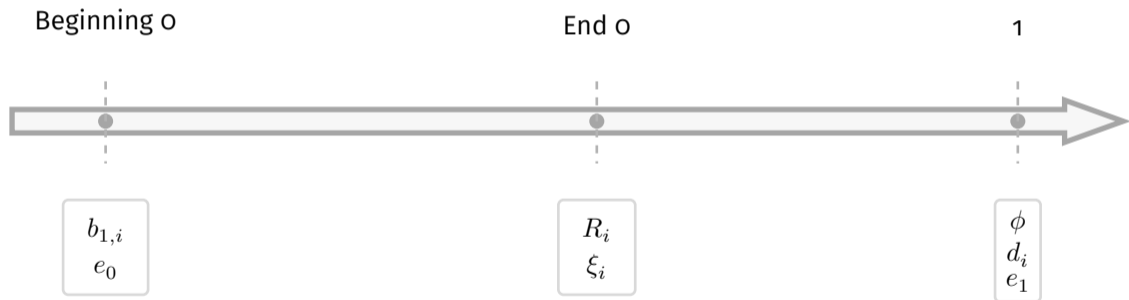
- Countries

- * Nominal rigidities in non-tradable goods

- * Government lacks commitment and can default on its debt

- * Possibility of multiple equilibria as in Calvo (1988)

Timeline



Households

- Preferences

$$U(c_0) - \nu(h_0) + \beta \mathbb{E}[U(c_1) - \nu(h_1)]$$

$$c = \left[\omega (c^T)^{-\mu} + (1 - \omega) (c^N)^{-\mu} \right]^{\frac{1}{-\mu}}$$

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- Budget constraint

$$e_t c_t^T + P_t^N c_t^N = e_t y^T + \Pi_t + W_t h_t + e_t T_t \text{ for all } t$$

Firms

- Linear production: $y^N = n$
- Sticky price: $P_t^N = \bar{P}^N$
- Given that prices are sticky, firms produce to satisfy demand: $n_t = c_t^N$

Government

- Fiscal budget:

$$R_0 b_0 + T_0 = b_1$$

$$(1 - d)R_1 b_1 = T_1(s)$$

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- Default cost (Ayres et al. (2023)):
 - * $\phi \in \{\phi_L, \phi_H\}$
 - * with probability ρ , $\phi = \phi_L$

International Lenders

- Excess returns:

$$M(R_1) = \mathbb{E}[R_1(1 - d) - R^*]$$

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Consequence of a bimodal default shock

Monetary Authority

- Preferences of the central bank

$$U = \int_i [u(c_{i,0}) - \nu(h_{i,0}) + \beta \mathbb{E}(u(c_{i,1}(s)) - \nu(h_{i,1}(s)) - \phi d_i(s))] di$$

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- Stochastic structure
 - * Aggregate shock: Default cost (ϕ)
 - * Idiosyncratic shock: a fraction π get the "bad" sunspot

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 - \downarrow
 - $\{0, \pi, 1\}$

Labor Market

- Equilibrium labor supply

$$\mathcal{H}(c_t^T, e_t) = \left[\frac{1 - \omega}{\omega} \frac{e_t}{\bar{P}^N} \right]^{\frac{1}{1+\mu}} c_t^T$$

Labor Market

- Equilibrium labor supply

CB can stabilize employment

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- Wedge

$$\tau_t(c_t^T, e_t) = u_N(c^T, \mathcal{H}(c_t^T, e_t)) - \nu'(\mathcal{H}(c_t^T, e_t))$$

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= 0 with flexible prices

Optimal Policy

Default Policy

- Value function:

$$V_1(R_1 b_1, \phi, e_1) = \max_d (1 - d)V^R(R_1 b_1, e_1) + dV^D(\phi, e_1)$$

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$$V_1(R_1 b_1, \phi, e_1) = \max_d (1 - d)V^R(R_1 b_1, e_1) + dV^D(\phi, e_1)$$

- Threshold:

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function of e_1 through $\mathcal{H}(c^T, e_1)$

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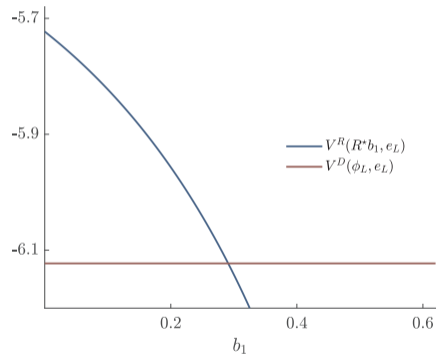
↓
function of e_1 through $\mathcal{H}(c^T, e_1)$

- Probability of default:

$$F(R_1 b_1, e_1) = Pr[\phi < \bar{\phi}(R_1 b_1, e_1)]$$

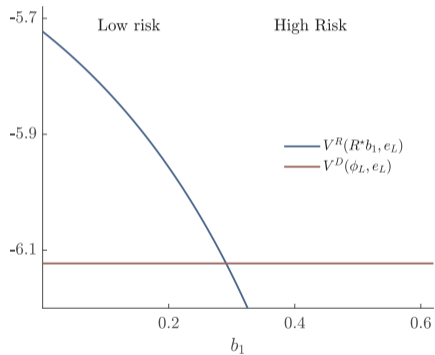
Multiplicity

Low e_1



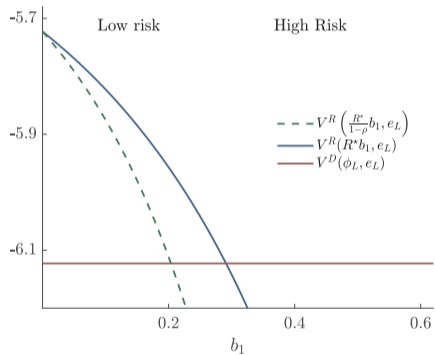
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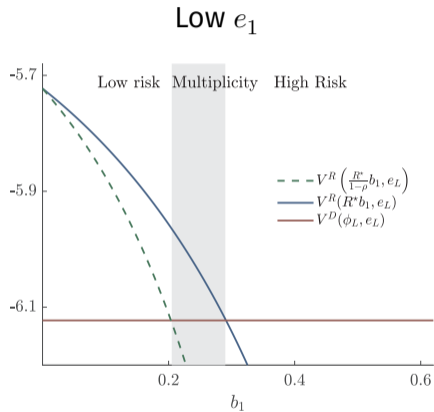


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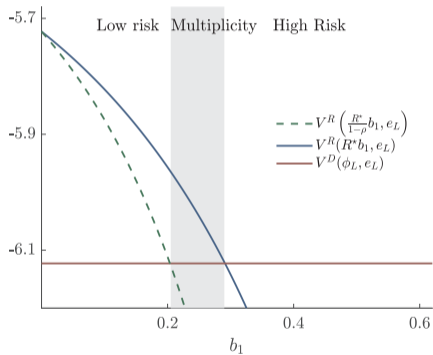
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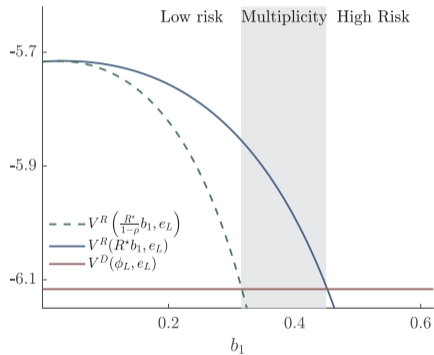
Lemma: There exist two thresholds of debt $b^-(e_1), b^+(e_1)$

Multiplicity

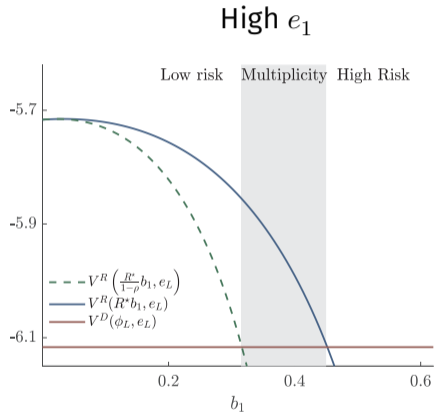
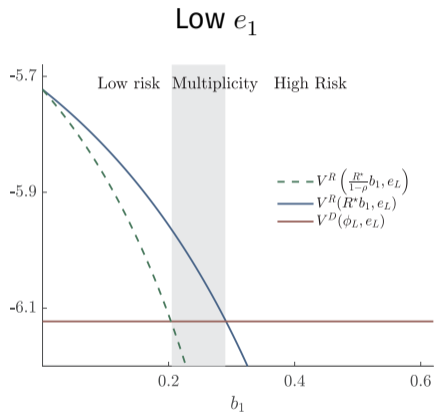
Low e_1



High e_1



Multiplicity



Proposition: Thresholds are increasing in $e_1 \Rightarrow$ Related to Bianchi and Mondragon (2022)

Monetary Policy in Period One

- Preferences of the CB

$$W(s, \phi) = \max_{e_1} \int_i [V_1(R_{1,i} b_{1,i}, e_1)] di$$

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CB close the average output gap ←

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CB close the average output gap ←

- CB face trade-off in monetary policy when there is heterogeneity

Optimal Policy in Period Zero

- Governments

$$V_0(e_0, s) = \max_{b_{1,i}} u(c_0^T, \mathcal{H}(c_0^T, e_0)) - \nu(\mathcal{H}(c_0^T, e_0)) + \beta \mathbb{E}[V_1(R_1 b_1, e_1(s, \phi))]$$

subject to

$$c_0^T = y^T + b_1 - R_0 b_0$$

$$R^* = [1 - F(R_1 b_1, e_1(s, \phi))] R_1$$

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Interest rate depends on the $\mathbb{E}[e_1(s, \phi)]$

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Countries choose same debt

» Markov Equilibrium

Optimal Policy Under Commitment

- Governments

$$V_0^C(e_0, e^C(\phi)) = \max_{b_1} u(c_0^T, \mathcal{H}(c_0^T, e_0)) + \beta \mathbb{E} \left[V_1^C(R^C(b_1, \xi, e_1) b_1, \phi, e^C(\phi)) \right]$$

subject to

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Interest rate now depends on promise $e^C(\phi)$

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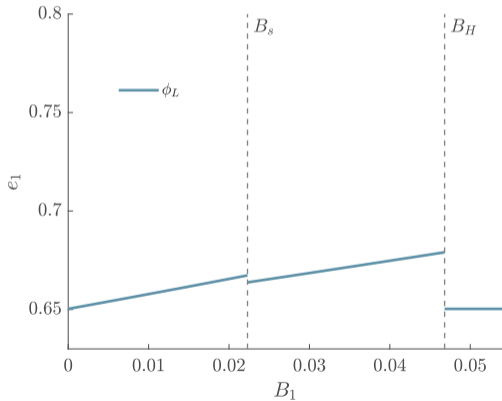
- Central bank

$$W^C = \max_{e_0, e^C(\phi)} \int_i \left[V_0^C(e_0, e^C(\phi)) \right] di$$

Optimal Monetary Policy

- Let $B_s = b^+$, $B_H = b^-$. For $B_1 < B_s$, e_M solves:

$$0 = \tau_t \left(y^T - R^* B_1, e_M \right)$$



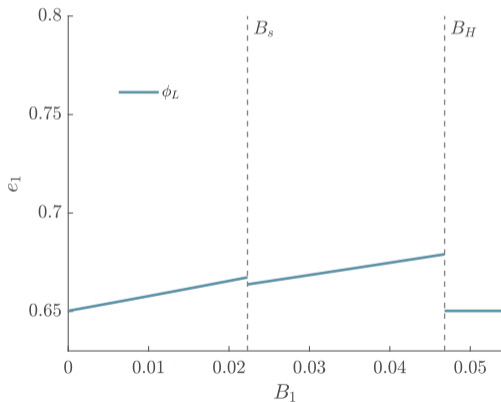
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- For $B_1 \in (B_s, B_H)$, $e_M(\phi_L)$ and $e_M(\phi_H)$ solves:

$$0 = \pi \tau_t \left(y^T, e_M(\phi_L) \right) \mathcal{H}_e \left(y^T, e_M(\phi_L) \right) \\ + (1 - \pi) \tau_t \left(y^T - R^* B_1, e_M(\phi_L) \right) \mathcal{H}_e \left(y^T - R^* B_1, e_M(\phi_L) \right)$$



Optimal Monetary Policy

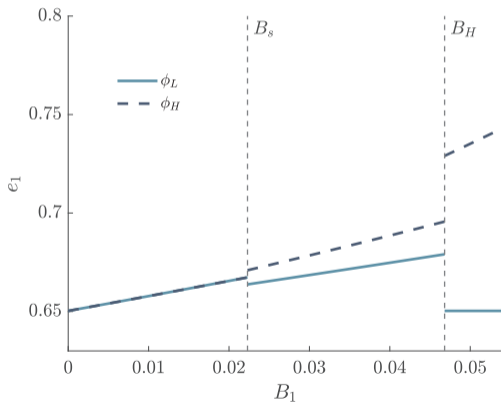
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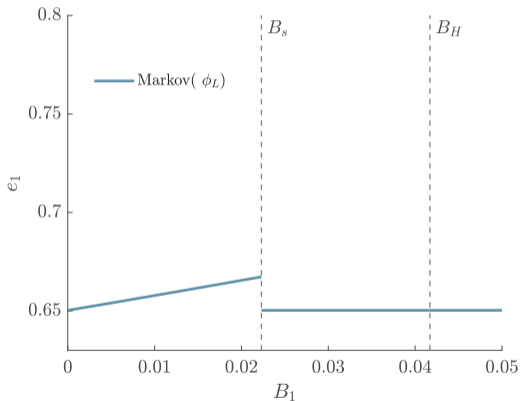
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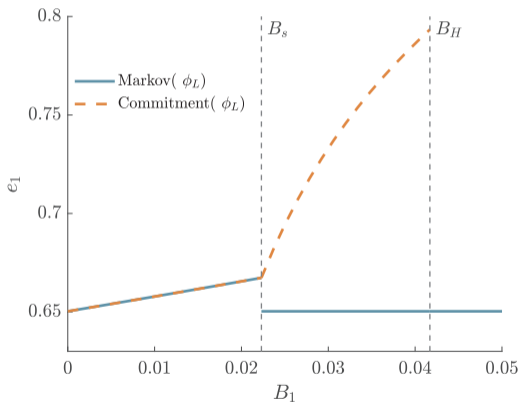
Commitment and Confidence Risk

- For $\pi = 1$, $\phi = \phi_L$ and $B_1 \in (B_s, B_H)$, $e_M(\phi_L)$ solves:

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Commitment and Confidence Risk



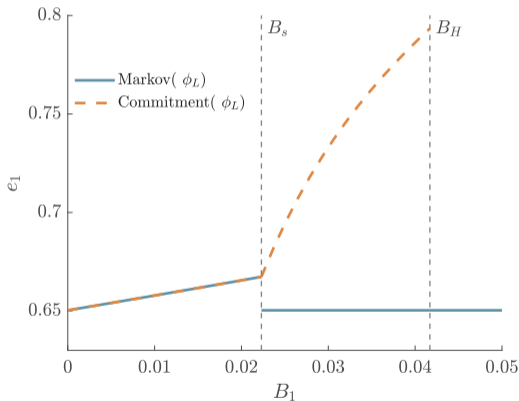
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$$V^D \left(\phi_L, e^C(\phi_L) \right) = V^R \left(\frac{R^*}{1 - \rho} B_1, e^C(\phi_L) \right)$$

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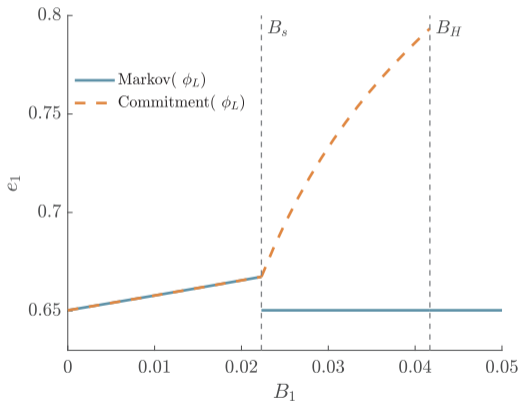
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- With $e^C(\phi_L)$:

$$R_i = R^* \text{ for all } i$$

Commitment and Confidence Risk



- For $\pi = 1$, $\phi = \phi_L$ and $B_1 \in (B_s, B_H)$, $e_M(\phi_L)$ solves:

$$0 = \tau \left(y^T, e_M(\phi_L) \right)$$

- Under commitment the CB choose $e^C(\phi_L)$:

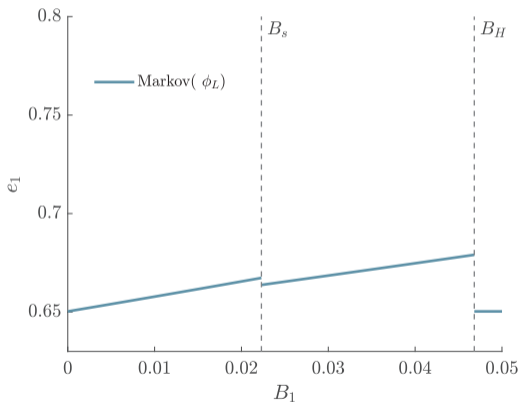
$$V^D \left(\phi_L, e^C(\phi_L) \right) = V^R \left(\frac{R^*}{1 - \rho} B_1, e^C(\phi_L) \right)$$

- With $e^C(\phi_L)$:

$$R_i = R^* \text{ for all } i$$

$$\tau \left(y^T - R^* B_1, e^C(\phi_L) \right) \neq 0$$

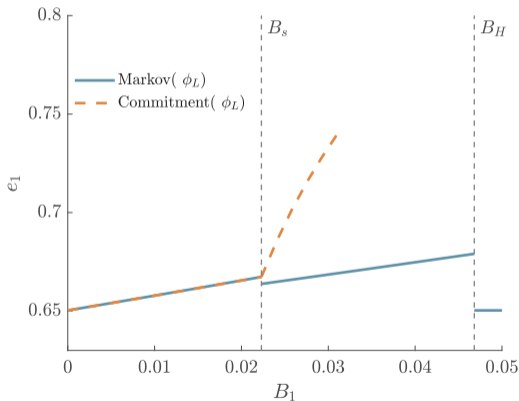
Commitment with Heterogeneity



- For $\phi = \phi_L$ and $B_1 \in (B_s, B_H)$, e_M solves:

$$0 = \pi \tau_t(y^T, e_M) \mathcal{H}_e(y^T, e_M) + \\ + (1 - \pi) \tau_t(y^T - R^* B_1, e_M) \mathcal{H}_e(y^T - R^* B_1, e_M)$$

Commitment with Heterogeneity



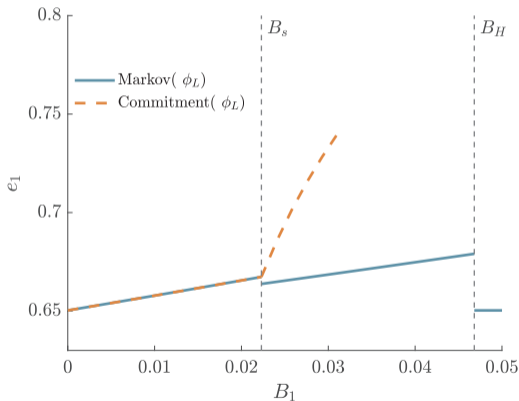
- For $\phi = \phi_L$ and $B_1 \in (B_s, B_H)$, e_M solves:

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- Under commitment the CB chooses $e^C(\phi_L)$ such that :

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Commitment with Heterogeneity



- For $\phi = \phi_L$ and $B_1 \in (B_s, B_H)$, e_M solves:

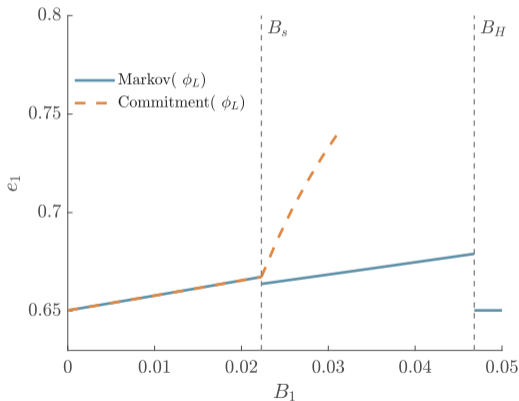
$$0 = \pi \tau_t (y^T, e_M) \mathcal{H}_e (y^T, e_M) + (1 - \pi) \tau_t (y^T - R^* B_1, e_M) \mathcal{H}_e (y^T - R^* B_1, e_M)$$

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- A fraction π of countries get lower interest rate

Commitment with Heterogeneity



- For $\phi = \phi_L$ and $B_1 \in (B_s, B_H)$, e_M solves:

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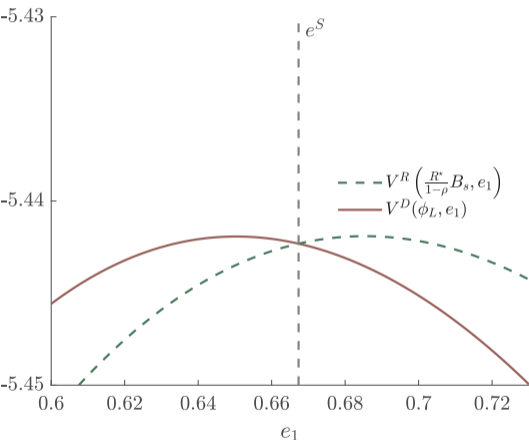
- A fraction π of countries get lower interest rate
- Strictly positive welfare gains from commitment only near the threshold

Taking Stock

- Markov equilibrium:
 - * Unique monetary policy
 - * For some debt, countries face idiosyncratic confidence risk
 - * Commitment of monetary policy can eliminate idiosyncratic confidence risk
- Next we will describe multiplicity in the Markov equilibrium
 - * For some debt, monetary union subject to confidence risk
 - * Multiple monetary policy
 - * In the "bad" Markov equilibrium region with idiosyncratic confidence risk expands
 - * Commitment of monetary policy can select the good Markov equilibrium

Endogenous Fragmentation

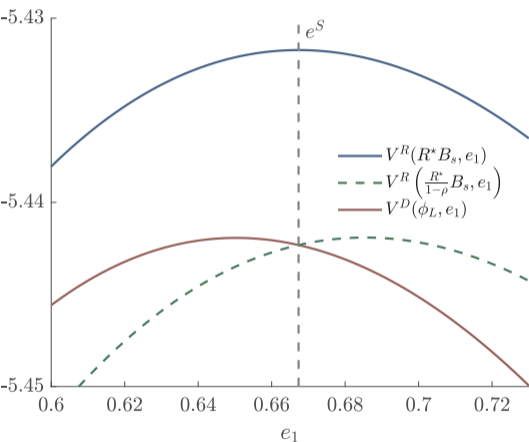
- Safe Equilibrium



- * Assume $B_1 = B_s$ (in the threshold)
- * $d_i = 0$ for all countries
- * $R_{i,1} = R^*$ for all countries
- * $\tau(c^T, e^S) = 0$ for all countries

Endogenous Fragmentation

- Safe Equilibrium



- * Assume $B_1 = B_s$ (in the threshold)
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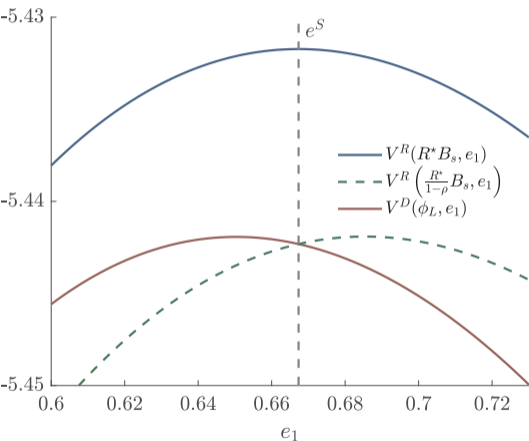
Endogenous Fragmentation

- Safe Equilibrium

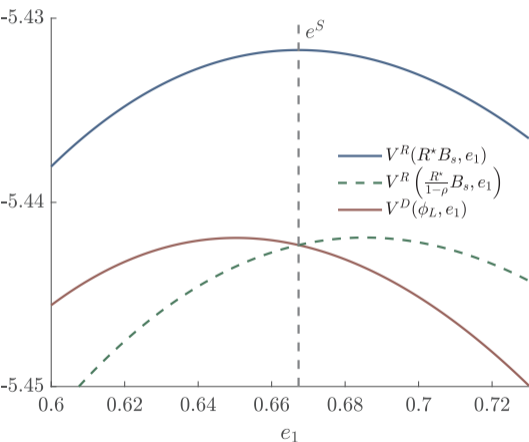
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- Fragmented Equilibrium

- * Guess $d_i = 1$ if $\phi = \phi_L$
- * $R_{i,1} = \frac{R^*}{1-\rho}$ for π countries



Endogenous Fragmentation



- Safe Equilibrium

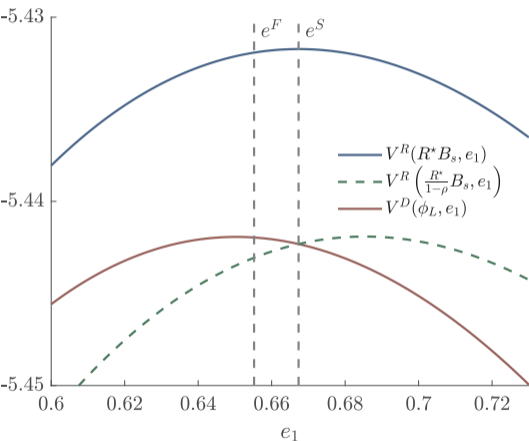
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- Fragmented Equilibrium

- * Guess $d_i = 1$ if $\phi = \phi_L$
- * $R_{i,1} = \frac{R^*}{1-\rho}$ for π countries
- * Full employment not feasible:

$$0 = (1 - \pi) \left[\tau_t \left(y^T - B_s R^*, e_F \right) \mathcal{H}_e \left(y^T - B_s R^*, e_F \right) \right] \\ + \pi \left[\tau_t \left(y^T, e_F \right) \mathcal{H}_e \left(y^T, e_F \right) \right]$$

Endogenous Fragmentation



- Safe Equilibrium

- * Assume $B_1 = B_s$ (in the threshold)
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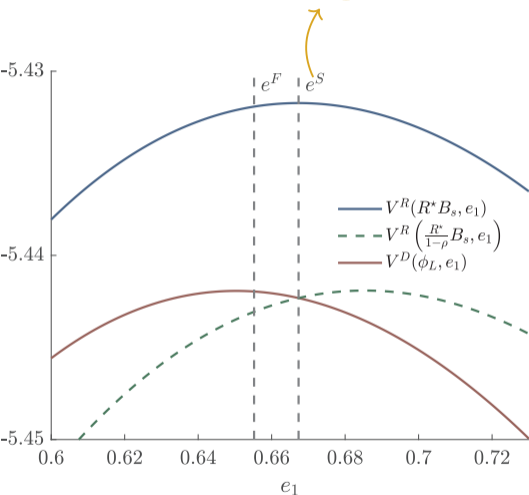
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Endogenous Fragmentation

Coincide with optimal e_1 under commitment



- Safe Equilibrium

- * Assume $B_1 = B_s$ (in the threshold)
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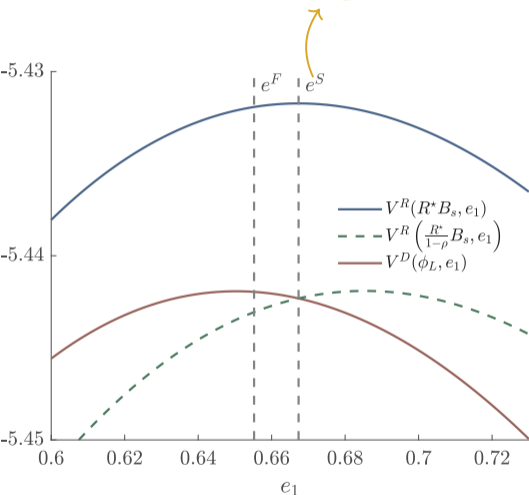
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Endogenous Fragmentation

Coincide with optimal e_1 under commitment



Related result to Fornaro and Grosse-Steffen (2024)

- Safe Equilibrium

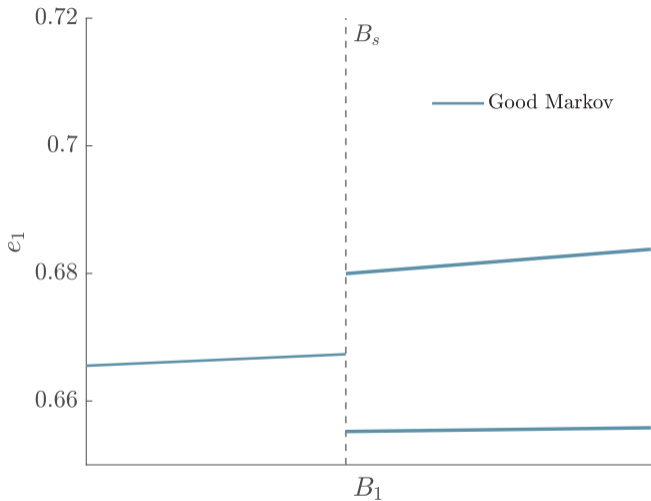
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- Fragmented Equilibrium

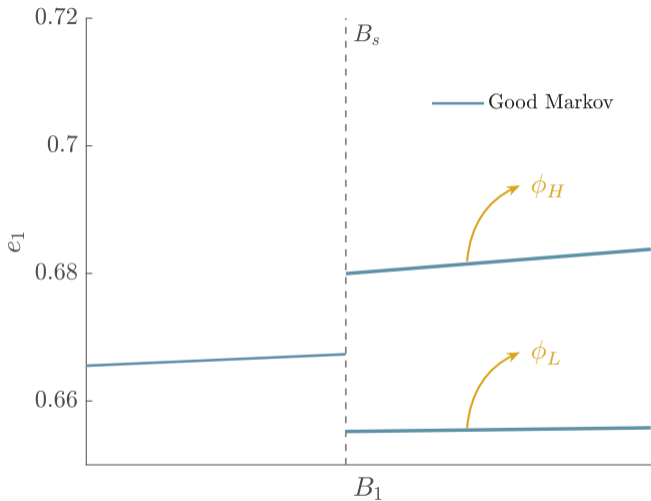
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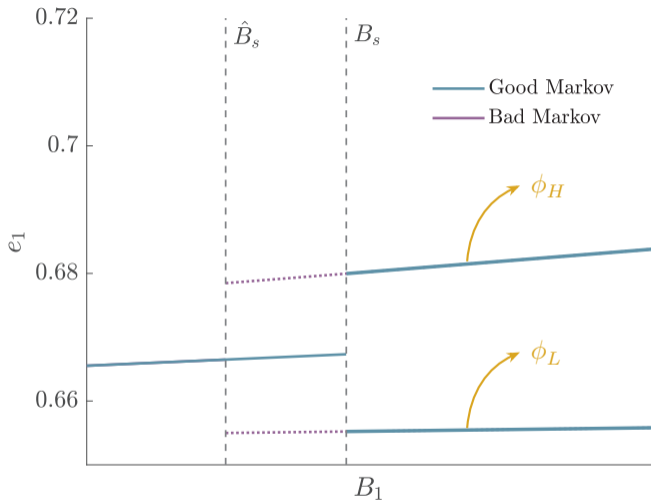
Optimal Monetary Policy



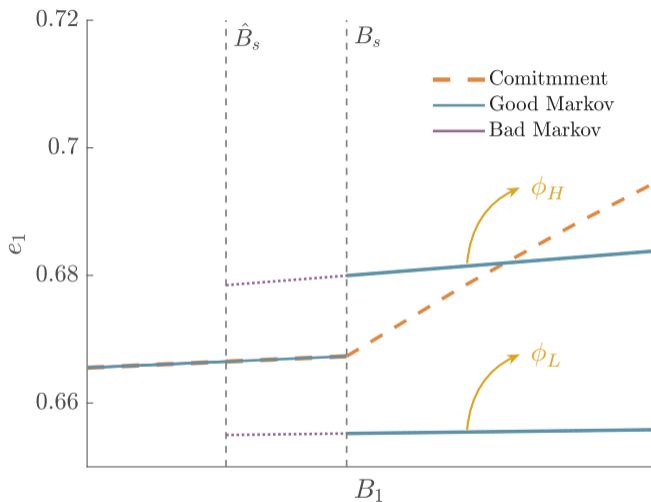
Optimal Monetary Policy



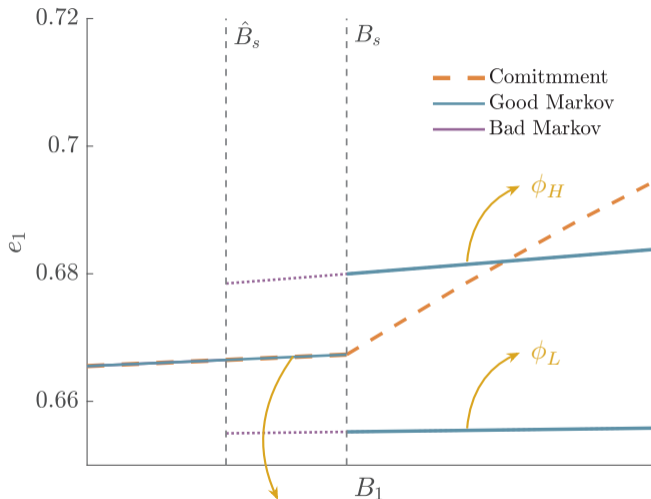
Optimal Monetary Policy



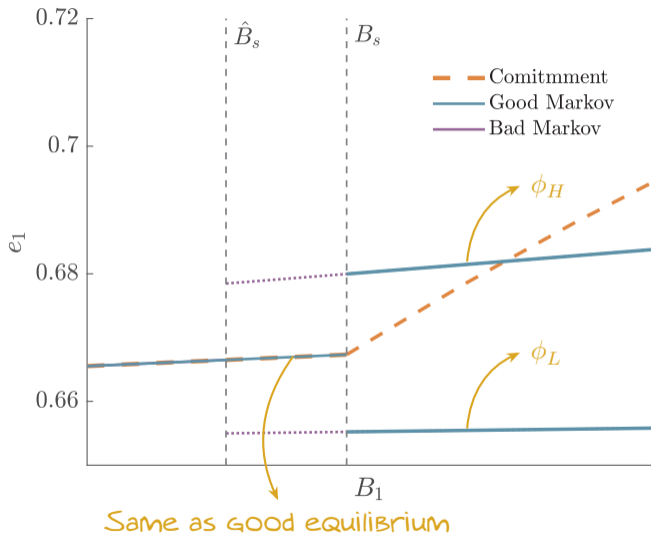
Optimal Monetary Policy



Optimal Monetary Policy



Optimal Monetary Policy



- In the multiplicity region commitment is a selection device

Conclusion

- Develop a model of a monetary union with self-fulfilling debt crisis
- Fragmented equilibrium emerges endogenously
 - * Heterogenous conditions imply that central bank unable to achieve full employment
 - * Rationalizes high spreads for some countries
 - * A commitment to running the economy hot can help ensure a safe equilibrium for all countries.

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- Bianchi, J. and Mondragon, J. (2022). Monetary independence and rollover crises. *The Quarterly Journal of Economics*, 137(1):435–491.
- Calvo, G. A. (1988). Servicing the public debt: The role of expectations. *American Economic Review*, 78(4):647–661.
- De Grauwe, P. (2013). The european central bank as lender of last resort in the government bond markets. *CESifo economic studies*, 59(3):520–535.
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<http://krugman.blogs.nytimes.com/2015/07/05/austerity-arithmetic/>.
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Competitive Equilibrium

Definition

(Competitive Equilibrium) Given an initial condition $\{R_0 b_0\}$; the equilibrium consists of a sequence of prices $\{R_1, e_t, P_t^N\}_{t=0,1}$; government policies $\{b_1, d(s), T_0, T_1(s)\}$, and households allocations $\{c_t^T, c_t^N, h_t\}_{t=0,1}$ such that:

- i Given prices and government policies: $\{c_t^T, c_t^N, h_t\}_{t=0,1}$ maximizes problem.
- ii Given prices and government policies: no arbitrage condition holds. That is, $M(R) = 0$
- iii Given prices and government policies: labor market clears $h_t = n_t$.
- iv Given prices and households policies: $\{b_1, d(s), T_0, T_1(s)\}$ are consistent with government budget constraints

Markov Equilibrium

Definition

(Markov Equilibrium) Given initial conditions $\{R_0 b_0\}$; a Markov perfect equilibrium is defined by a set of strategies profile $\{\mathcal{E}_0, \mathcal{B}_1, \mathcal{E}_1(Rb_1, \phi), \mathcal{D}_1(Rb_1, \phi)\}$; value functions $\{V_0, V_1(Rb_1, \phi)\}$ and an interest rate schedule $\mathcal{R}(b_1)$ such that:

- i Given the interest rate schedule: $\{\mathcal{E}_0, \mathcal{B}_1\}$ solves the government problem in period zero, and V_0 attains the maximum.
- ii Given a face value of debt; $\{\mathcal{E}_1(Rb_1, \phi), \mathcal{D}_1(Rb_1, \phi)\}$ solves the government problem at every state in period one, and $V_1(Rb_1, \phi)$ attains the maximum.
- iii Given the policy functions of the government: $\mathcal{R}(b_1)$ satisfies the break-even condition of international investors.

back

Markov Equilibrium - Monetary Union

Definition

(Symmetric Markov Equilibrium) Given initial conditions $\{R_0 b_0\}$; a Markov perfect equilibrium is defined by a set of strategies profile $\{\mathcal{E}_0, \mathcal{E}_1(s), \{\mathcal{B}_{i,1}, \mathcal{D}_{i,1}(R_i b_{i,1}, s)\}_i\}$; value functions $\{W, \{V_{i,0}, V_{i,1}(R_i b_{i,1}, s)\}_i\}$ and a set interest rate schedules $\{\mathcal{R}_i(b_{i,1}, s, \xi)\}$ such that:

- i Given the interest rate schedule and the policy function of the central bank: $\{\{\mathcal{B}_{i,1}\}$ solves the problem of each government in period one, and $\{\{V_{i,0}\}_i\}$ attains the maximum.
- ii Given a face value of debt and the policy function of the central bank: $\{\{\mathcal{D}_{i,1}(R_i b_{i,1}, s)\}_i\}$ solves the problem of each government at every state, and $\{\{V_{i,1}(R_i b_{i,1}, s)\}_i\}$ attains the maximum.
- iii Given the policy functions of the governments and the interest rate schedules: $\{\mathcal{E}_0, \mathcal{E}_1(s)\}$ solves the problem of the central bank, and W attains the maximum.
- iv Given the policy functions of the government and the central bank, \mathcal{R}_i satisfies the break-even condition of international investors for each country.

back

Interest Rate Schedule

- We assume the sunspot works as follows:

$$R_1(b_1, \xi) = \begin{cases} R^* & b_1 < b^+ \\ R^* & b^- > b_1 \geq b^+ \text{ with prob. } \pi \\ \frac{R^*}{1-\rho} & b^- > b_1 \geq b^+ \text{ with prob. } (1-\pi) \\ \frac{R^*}{1-\rho} & b_1 \geq b^- \end{cases}$$

- Government face uncertainty in the interest rate if chooses $b \in \mathcal{I}^R$

Optimal Debt



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Multiple Interest Rate Schedule

- Optimistic

$$\hat{R}_{MU}(b_1, e_1(s), \xi) = \begin{cases} R^* & b_1 < \hat{b}_{MU}^+(s^S) \\ R^* & \hat{b}_{MU}^-(e_1(s)) > b_1 \geq \hat{b}_{MU}^+(s^S) \text{ with prob. } \pi \\ \frac{R^*}{1-\rho} & \hat{b}_{MU}^-(e_1(s)) > b_1 \geq \hat{b}_{MU}^+(s^S) \text{ with prob. } (1 - \pi) \\ \frac{R^*}{1-\rho} & b_1 \geq \hat{b}_{MU}^-(e_1(s)) \end{cases}$$

- Pessimistic

$$\bar{R}_{MU}(b_1, e_1(s), \xi) = \begin{cases} R^* & b_1 < \bar{b}_{MU}^+(s^H) \\ R^* & \bar{b}_{MU}^-(e_1(s)) > b_1 \geq \bar{b}_{MU}^+(s^H) \text{ with prob. } \pi \\ \frac{R^*}{1-\rho} & \bar{b}_{MU}^-(e_1(s)) > b_1 \geq \bar{b}_{MU}^+(s^H) \text{ with prob. } (1 - \pi) \\ \frac{R^*}{1-\rho} & b_1 \geq \bar{b}_{MU}^-(e_1(s)) \end{cases}$$

Optimal Debt and Multiplicity

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Timing

- Timing:

