Domestic Debt and Self-Fulfilling Crises *

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Abstract

This paper shows the vulnerability of the government to face a self-fulfilling crisis characterized by low domestic demand for government bonds associated with a high probability of default. Investor expectations of low domestic demand increase foreign debt and the probability of default. The high risk of default decreases bond prices and leads to higher taxes or reduced government transfers. With low liquidity due to higher taxes, domestic demand is low, confirming the initial expectation. I introduce a version of a sovereign debt model with domestic and foreign investors, analyze conditions for multiple equilibria, and establish that the government must announce a sequence of subsidies contingent on aggregate demand to restore efficiency.

Keywords: Self-fulfilling debt crisis, sovereign debt, multiple equilibria, default risk, debt sustainability

JEL classification: F21,F61, E62, F64, F34, F38, F41, P41, P43.

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1 Introduction

In the aftermath of the European debt crisis, a growing body of literature has studied macroeconomic conditions that make an economy vulnerable to episodes of self-fulfilling crises. The primary concern of policymakers at that time was that high interest payments undermined fiscal stability and exacerbated risk, thereby becoming self-fulfilling. Consequently, efforts were focused on mitigating confidence-driven fluctuations in spreads. Another important point of discussion was the role of the domestic market in increasing or mitigating default risk. As domestic investors are the most important lender for the government, there was concern that fiscal consolidation could, in fact, reduce the domestic demand for debt and further reduce the confidence of the market in sovereign bonds.

In this complex environment, the key trade-off for governments was whether to increase borrowing to manage soaring interest payments or pursue fiscal consolidation, which could deepen the crisis and decrease the capacity of the domestic market to absorb the new government debt. This phenomenon gives rise to a new set of questions: How does the composition of the bondholder influence the incentives for governmental debt repayment? How does it shape the formation of market expectations about future risk? And what role does it play in increasing the government's vulnerability to self-fulfilling crises? Can financial regulation reduce the risk of self-fulfilling crisis?

To address these questions, this paper introduces a version of the canonical model of sovereign default with domestic and foreign investors. Analytically, I solve for equilibrium objects and study conditions under which the model supports multiple equilibria. I establish that when the economy is vulnerable to self-fulfilling crises, the government needs to announce a sequence of policies contingent on all possible realizations of the domestic demand for bonds to restore efficiency. The main result of the paper concerns the existence of a novel type of multiplicity in sovereign debt models due to the presence of domestic and foreign investors. In particular, I show that without financial regulation, the government is exposed to a type of self-fulfilling crisis where the domestic demand for bonds is low, and as a consequence, the risk of default is high. Two assumptions are key for multiplicity. The government is benevolent, so it has lower incentives to default when bondholders are domestic investors.

Second, fiscal adjustments such as higher taxes or lower transfers from the government tighten the budget constraint of domestic investors, which affects their demand for government bonds.

I focus on a two-period model in which the only uncertainty is an exogenous utility cost that the government faces when defaulting in the second period. Domestic investors receive an exogenous endowment, pay taxes (or receive transfers), and can only save using government bonds. The government is benevolent, so it has the same preferences as domestic investors. In addition, the government's initial debt and new borrowing are exogenous and fixed.

A key feature of the model is that the government cannot control who buys the debt. I consider a government that issues debt in a single market populated by domestic and foreign investors. In contrast to Eaton and Gersovitz (1981), the level of foreign debt becomes an equilibrium outcome rather than a choice of the government. However, because payments to foreign investors reduce domestic resources, the risk of default depends on the level of foreign debt, as in Eaton and Gersovitz (1981). It implies that for a given level of borrowing, the price of government bonds depends on investors' expectations about the domestic demand for bonds. I derive an analytical condition for preferences and the distribution of the shocks under which this environment supports multiple equilibria.

The multiplicity in the model is static. That is, expectations of future behavior are constant and the market price for government bonds is still indeterminate. The main force driving multiplicity is the strategic complementarities between domestic investors. When the aggregate domestic demand for bonds is high, the demand for government bonds from individual domestic investors increases. The mechanism is as follows. Given a level of government debt, investors anticipate a higher foreign debt if they expect low domestic demand for bonds. Higher foreign debt induces a higher probability of default, reducing the prices of sovereign debt and the government's resources. Consequently, the government has to reduce the deficit by increasing taxes or reducing transfers. Fiscal adjustment tightens the budget constraints of domestic investors, reducing their demand for government bonds, and confirming pessimistic beliefs. On the other hand, if agents expect high domestic demand for bonds, the price is high, there are more resources in the economy, domestic investors buy high shares of government debt, and the risk of default is low, which confirms the initial expectation.

As long as the price elasticity of sovereign bonds to the external debt is high enough for some level of borrowing, the model can exhibit multiple equilibria. Both the level of debt and the value of new borrowing are essential for indeterminacy, which arises only for a high level of new borrowing. Conversely, when borrowing is sufficiently low, the risk is low, so price changes due to changes in the composition of bondholders are mild, and the model behaves like the standard Eaton-Gersovitz model with one single equilibrium. On the other hand, indeterminacy arises only for intermediate values of outstanding debt. When the initial debt is too high, taxes are always high and there is only one equilibrium in which the domestic demand for bonds is low. In addition, when the initial debt is low enough, taxes are low, and the domestic demand for bonds is always high in equilibrium.

Another key result of the model is that the domestic demand for government bonds is inefficiently low. Domestic investors do not internalize the effect of increasing their demand on the government's incentives to default, so their demand is inefficiently low. I solve the problem of a social planner who can directly choose the level of foreign debt as in Eaton-Gersovitz, compare the solution with the decentralized equilibrium, and show there is space for policy intervention. The mechanism is as follows. For a given level of borrowing, low domestic demand for government bonds induces higher foreign debt, a high risk of default, and an inefficiently low price of government bonds.

This mechanism contrasts with a related result in the literature on financial repression; see, for example, Chari et al. (2020), which argues that the domestic demand for government bonds is inefficiently low because it brings discipline to the government. In this view, when the government cannot selectively default on some investors, a default would imply disruptions in the domestic financial system and capital investment. As a consequence, a high level of domestic debt increases the cost of default and increases the price of the bonds. One key implication of the difference in the mechanism of inefficiency is that, in my model, there is scope for policy intervention even if the government can selectively default on foreign investors. A second contribution to this literature is that I analyze how inefficiency could be boosted at the aggregate level, generating the possibility of having multiple equilibria.

The existence of multiple equilibria provides a theory of financial regulation. I study a subsidy on the domestic demand for bonds. I show that, in the presence of the risk of self-fulfilling crises, the government must announce a sequence of subsidies contingent on all possible realizations of the domestic demand for bonds. This announcement helps the government align investors' expectations about domestic demand and prevents self-fulfilling crises.

One key implication of the multiplicity for the literature on financial repression is that policy announcements are key for implementing efficient equilibrium because they help shape expectations. Another implication is that state-contingent policies cannot restore efficiency. Instead, the government must announce a sequence of policies contingent on the domestic demand for bonds to align expectations.

The model offers at least three lessons for the sovereign debt literature. First, the presence of heterogeneous investors could generate strategic complementarities and be a novel source of multiplicity in models with default risk. Second, in a model with foreign and domestic investors, the level of foreign debt is not, in general, efficient. Third, implementing the constrained-efficient levels of foreign debt in the spirit of Eaton and Gersovitz (1981) requires a complex set of policies that help the government manage expectations and prevent self-fulfilling crises.

Related Literature. There are two canonical examples of multiplicity in sovereign debt models, namely Cole and Kehoe (2000) and Calvo (1988)[1]. Similarly to Cole and Kehoe (2000), the multiplicity in my model is static. While Cole and Kehoe (2000) explores the role of timing assumptions in the possibility of having multiple equilibria, my analysis focuses on the presence of domestic investors in the market and the feedback loop between government debt prices and domestic demand for bonds as factors that generate multiple equilibria. To shed light on this novel mechanism, I adopt the Eaton-Gersovitz timing and show that the model supports multiple equilibria. For further research on Cole-Kehoe-type runs, see Bocola and Dovis (2019), Conesa and Kehoe

[1]See Ayres et al. (2018) and Bianchi and Mondragon (2022) for further work.

(2017), and Bianchi and Mondragon (2022).

As in my model, the feedback between prices and the budget set is key to explaining the multiplicity in Calvo (1988). The mechanism in Calvo can be easily understood in an environment where the government is forced to raise a certain amount of debt. Given a level of debt, high interest rates increase debt payments in the future, increasing incentives to default. Moreover, the resulting high probability of default supports the initial high interest rate. This feedback loop results in an equilibrium with high interest rates and default risk. In contrast, lower interest rates require lower debt payments, reducing incentives to default. Consequently, it is possible to construct another equilibrium characterized by lower interest rates and a low risk of default. Ayres et al. (2018) explores this mechanism in a model with the risk of having long periods of economic crisis.

Galli (2021) analyzes how fiscal responses to low bond prices could generate a reinforcing loop that increases the risk of default. In contrast to my model, Galli explores the feedback between capital investment and default risk. Similarly to my environment, when the government faces low prices due to pessimistic expectations of foreign investors, it responds by increasing taxes. This increase in taxes reduces the optimal capital investment of domestic firms. In the presence of a cost of default on productivity, this reduction in capital investment increases the probability of default, confirming the initial pessimistic expectations.

Broner et al. (2014) also explores multiplicity in an economy with capital investment. They analyze a model with domestic and foreign investors and study a case of multiple equilibria. In sharp contrast to my result, they argue that high domestic holdings of government debt could trigger a high risk of default by crowding out capital investment. I see their mechanism as complementary to the one described in this paper. It is an open quantitative question to explore a model in which both mechanisms operate and establish conditions under which the positive effect of lower foreign debt due to higher domestic demand for bonds dominates the negative impact of crowding out investment. Another critical difference with my model is that, in their environment, the government's ability to default on international investors selectively is key to delivering multiplicity. Instead, in my model, the existence of multiple equilibria does not depend on whether the government can selectively default on foreign investors.

Other important contributions to the literature on self-fulfilling crises are Aguiar and Amador (2020), Lorenzoni and Werning (2019), and Passadore and Xandri (ming), which analyze how expectations of future behavior could generate sell-fulfilling dynamics. The key friction considered in Lorenzoni and Werning (2019) is that the government cannot adjust taxes in response to fluctuations in bond prices and is forced to collect a certain amount of resources through debt. If investors expect a high probability of default, the prices of bonds drop. Low bond prices force the government to issue more debt to meet its fiscal needs, increasing the risk of default, and confirm the initial expectation. In contrast, in my model, the government can adjust taxes in response to low bond prices. I emphasize that the option of adjusting taxes might not eliminate the possibility of multiple equilibria because it has an impact on the domestic demand for government bonds, which ultimately affects default risk and makes the government vulnerable to self-fulfilling crises.

This paper also relates to the literature that studies conditions for having unique equilibrium in sovereign debt models. I build on Aguiar and Amador (2019) and Auclert and Rognlie (2016) analyzing the features of my model that break the existing proof of uniqueness in the literature of sovereign default.

This paper belongs to the literature on the composition of bondholders in sovereign debt markets. This literature studies the efficiency of the bondholder composition of sovereign debt; some notable examples are Chari et al. (2020), and Mallucci (2022). Chari et al. (2020) analyses a model where the government cannot selectively default on one of the investors. Under this assumption, the government must consider the effect of a default in the budget constraint of domestic banks and the financial system. Consequently, if the government cannot selectively default, a higher domestic demand for bonds increases the cost of defaulting to the government and brings some discipline. Because domestic banks do not internalize this effect, this literature finds that their demand for government bonds is inefficiently low. This paper has at least two contributions to this literature. First, I show that in an open economy, there is a complementary mechanism that explains why domestic demand for bonds is inefficiently low. If the government should finance a certain amount of debt, and the domestic demand for bonds is low, the foreign debt increases, and the risk of default is inefficiently high. Critically, this novel mechanism is robust to whether the government can selectively default on some of its debt. Second, I analyze how inefficiencies in the domestic market can lead to multiple equilibria and expose the government to the risk of a self-fulfilling crisis. Second, I analyze the implications of multiplicity for the optimal design of financial regulation.

D Erasmo and Mendoza (2021) and Sunder-Plassmann (2020) also study sovereign default models with domestic and foreign investors. As in my model, they consider an economy where the government cares about domestic investors, so the risk of default depends on the bondholder composition.

This paper also relates to the literature on the default cost when bondholders are domestic banks. Some examples are Sosa-Padilla (2018), Perez (2018), and Arellano et al. (2019). Another notable example is Farhi and Tirole (2018), which studies how the cost of default on the domestic financial system and the government's bailout of domestic banks create the possibility of a doom loop between fiscal and financial crises.

Finally, this paper relates to Bassetto (2005), which analyzes the efficiency of state-contingent policies in the presence of multiplicity. I establish that state-contingent policies are ineffective in restoring efficiency when there are multiple equilibria. In the model, the government must announce a sequence of policies contingent on all possible realizations of domestic demand for government bonds. The announcement of a sequence of policies helps the government align the expectations of private investors and select the best equilibria.

Outline. The paper is organized as follows. Section 2 presents the model. Section 3 describes the optimal government policy. Section 4 studies the multiplicity. Section 6 describes the constrained-efficient allocations. Section 7 discusses the policy implications of multiplicity. And Section 9 concludes.

2 Environment

I consider a small open economy that lasts for two periods, indexed by $t \in \{0, 1\}$. The key departure from standard frameworks concerns the presence of two types of investor. In particular, I consider an environment where the government issues debt in a single market populated by domestic and foreign investors. The government cannot control who buys its debt, and the law of one price applies. In addition, I assume that the government cannot selectively default on foreign investors. The timing is Eaton-Gersovitz, so the government commits to repay all outstanding debt before issuing new bonds.

2.1 Exogenous Processes.

Uncertainty is modeled by an exogenous utility cost (ν) incurred by governments when default occurs.

Assumption 1. The cost of default ν satisfies the following conditions: (i) $\nu \in \mathbb{V} \equiv (\tilde{\nu}, \underline{\nu})$; (ii) ν is drawn in the second period from a distribution independent of debt with pdf. $f(\nu)$.

This assumption is adopted for expositional convenience and could be easily replaced by uncertainty about endowments or government expenditures.

2.2 Domestic Investors.

The representative domestic investor has time-separable preferences with utility over consumption $\{c_t\}$ represented by

$$U = u(c_0) + \beta \mathbb{E}[u(c_1(\nu)) - d(\nu)\nu], \tag{1}$$

where $d \in \{0, 1\}$ takes the value of 1 if the government defaults or zero otherwise. The utility function u satisfies the following conditions: (i) $u : X \to \mathbb{R}$, where $X \equiv (0, \infty)$; and (ii) u is twice differentiable, with $u' > 0$ and $u'' < 0$.

The domestic investor cannot borrow and can only save using government bonds.

The government has access to lump sum taxes (if $T_0 > 0$) or transfers (if $T_0 < 0$). In addition, b_1 stands for the individual holdings of domestic investors in government debt, and q is the price of bonds. The budget constraint of the domestic investor in the first period is the following.

$$c_0 + qb_1 + T_0 = y \tag{2}$$

$$b_1 \geq 0. \tag{3}$$

In period one, domestic investors receive the returns on their savings only when the government does not default. At each ν, the budget constraint of the domestic investor becomes

$$c_1(\nu) + T_1(\nu) = y + b_1(1 - d(\nu)). \tag{4}$$

The domestic investors problem is to choose the sequences $\{c_0, \{c_1(\nu)\}_\nu, b_1\}$ to maximize (1), subject to (2), (3) and (4). B_1^D is the aggregate debt held by domestic investors.

2.3 International Lenders.

Similarly to Eaton and Gersovitz (1981), there is a continuum of identical foreign lenders that are risk neutral. They have access to a risk-free asset with a gross interest rate $\frac{1}{\beta}$. I also assume that they collectively have enough resources to buy any arbitrary number of government bonds. The asset pricing condition for bonds is therefore

$$q = \mathbb{E}\Big[1 - d(\nu)\Big]\beta. \tag{5}$$

2.4 Government.

The government enters with an initial foreign debt B_0 and fixed borrowing B_1. Then, in the first period, the government should adjust taxes (or transfers) to clear the fiscal balance. This is

$$B_0 + T_0 = qB_1. \tag{6}$$

In period one, in each ν, the government chooses whether to default, so the fiscal budget becomes

$$(1 - d(\nu))B_1 + T_1(\nu) = 0. \tag{7}$$

2.5 Competitive Equilibrium.

We can add the budget constraints of the domestic investor and the government to obtain the resource constraints of the economy.

$$c_0 + B_0 = y + q(B_1 - B_1^D) \tag{8}$$

$$c_1(\nu) = y - (1 - d(\nu))(B_1 - B_1^D). \tag{9}$$

Note that the resource constraints depended on the domestic demand for government bonds B_1^D. In the first period, the value of the foreign debt of the economy $q(B_1 - B_1^D)$ resembles a debt-laffer curve with an endogenous maximum. The main difference from Eaton and Gersovitz (1981) is that in this economy, foreign debt is an equilibrium outcome rather than a choice of the government. We are ready to define the competitive equilibrium of the small open economy.

Definition 1. (Competitive Equilibrium) Given a sequence of debt $\{B_0, B_1\}$ and government policies $\{T_0, \{T_1(\nu), d(\nu)\}_\nu\}$, an equilibrium consists of a price q, and domestic investor allocations $\{c_0, \{c_1(\nu)\}_\nu, b_1\}$ such that

i Given prices and government policies, $\{c_0, \{c_1(\nu)\}_\nu, b_1\}$ maximizes (1) subject to (2), (3) and (4);

ii Given government policies, q satisfies (5);

iii Given prices and domestic investor's allocations, $\{T_0, \{d(\nu), T_1(\nu)\}_\nu\}$ is consistent with government budget constraints (6) and (7);

iv Market clears: $B_1^D = b_1$.

3 Optimal Policy

In this section, I characterize the problem of the government. The government is benevolent, so it maximizes domestic investors' utility subject to resource constraints. Also, it lacks commitment, so I will focus on the Markov equilibrium in which all policies depend on the payoff-relevant states $\{\nu, B_1^D\}$.

I solve the model by backward induction. So, in the second period, the government decides its optimal policy for each pair of default cost (ν) and aggregate demand for government bonds (B_1^D). Next, domestic investors choose savings according to the government's policies. Finally, given the risk neutrality of international investors, I replace their problem with a break-even condition.

3.1 Problem of the Government.

The government value function in the second period is the following:

$$V(B_1^D, \nu) = \max_{d \in \{0,1\}} (1-d)u(y - B_1 + B_1^D) + d(u(y) - \nu). \tag{10}$$

The only choice for the government is to default or repay. Also, note that the value under repayment is an increasing function in the domestic holdings of government bonds, so the government has fewer incentives to default when the domestic debt is high. It is possible to characterize the default decision of the government by defining the following threshold:

$$\bar{V}(B_1^D) \equiv u(y) - u(y - B_1 + B_1^D). \tag{11}$$

This threshold represents the utility surplus derived from the higher consumption under default. Using this threshold, the default function of the government is

$$d(B_1^D, \nu) = \begin{cases} 1 & \text{if} \quad \nu < \bar{V}(B_1^D), \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

This condition means that if the default cost is lower than the surplus derived from higher consumption, the government defaults in the second period. The probability of default is

$$F(\bar{V}(B_1^D)) = \int_{\underline{\nu}}^{\bar{V}(B_1^D)} f(\nu) d\nu. \tag{13}$$

Finally, using the fiscal budget of the government, we can solve the tax policy for the second period as follows:

$$T_1(B_1^D, \nu) = \begin{cases} 0 & \text{if} \quad \nu < \bar{V}(B_1^D), \\ B_1 & \text{otherwise.} \end{cases} \tag{14}$$

3.2 Foreign Investors.

Given the probability of default, the break-even condition of international investors is as follows.

$$q(B_1^D) = \beta[1 - F(\bar{V}(B_1^D))]. \tag{15}$$

Using this price function and the budget constraint of the government in the first period, the tax policy in the first period is given by:

$$T_0(B_1^D) = B_0 - \beta[1 - F(\bar{V}(B_1^D))]B_1. \tag{16}$$

3.3 Problem of the Domestic Investors.

Next, I use government policies and price to define the problem of domestic investors as

$$W(B_1^D) = \max_{b_1} u(c_0) + \beta\mathbb{E}[u(c_1(B_1^D, \nu)) - d(B_1^D, \nu)\nu], \tag{17}$$

subject to

$$c_0 + q(B_1^D)b_1 + T_0(B_1^D) = y \tag{λ_1}$$

$$b_1 \geq 0. \tag{μ}$$

$$c_1(B_1^D, \nu) + T_1(B_1^D, \nu) = y + (1 - d(B_1^D, \nu))b_1 \tag{$\lambda_2(\nu)$}$$

Let μ be the multiplier of the borrowing constraint. Then, the FOCs for domestic investors are

$$0 = \mu\Big(q(B_1^D)u'(c_0(B_1^D)) - \beta[1 - F(\bar{V}(B_1^D))]u'(c_1^R(B_1^D))\Big) \tag{18}$$

$$0 = b_1\mu. \tag{19}$$

Where $c_0(B_1^D)$ and $c_1^R(B_1^D)$ stand for consumption in the first period and consumption in the second period in states where the government repays. Formally, I combine the budget constraint of the government and the fiscal budget to define:

$$c_0(B_1^D) = y + \beta[1 - F(\bar{V}(B_1^D))](B_1 - B_1^D) - B_0 \tag{20}$$

$$c_1^R(B_1^D) = y - B_1 + B_1^D. \tag{21}$$

Note that (18) and (19) are complementary slack conditions, which state that if domestic investors are not constrained–that is, $b_1 \neq 0$– then the Euler equation should hold. From the perspective of the individual domestic investors, given a price and a default rule of the government, the problem's constraints are a convex set. So, because of the concavity of the utility function, the solution to this problem is unique. However, at the aggregate level, the Euler equation could have multiple solutions.

The fact that the value of the domestic investors depends on the aggregate domestic demand for bonds speaks about the strategic complementarities operating in the model. On the one hand, the aggregate demand for bonds shapes the default function of the government in the second period. So, it affects the price at which domestic investors buy bonds and the future states at which they expect to be repaid. Note from (15) and (18) that these two effects cancel out given the risk neutrality of foreign investors and given that consumption is constant in states where the government repays. As a result, the Euler equation for domestic investors equates the marginal utility in the first and second periods under repayment.

In addition, the aggregate demand for government bonds affects the budget constraint of domestic investors in period zero. Changes in aggregate demand for bonds affect tax policy through its effect on bond prices. The key complementarity in the model is the effect of aggregate demand for bonds on the taxes (or transfers) in the first period. In the next section, I will focus on this effect to explain how complementarities can lead this economy to multiple equilibria.

3.4 Markov-Equilibrium.

The Markov equilibrium in this environment is defined as follows.

Definition 2. (Markov equilibrium) Given an initial debt B_0 and a government borrowing B_1, a perfect Markov equilibrium is defined by a set of strategies $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{B}_1, \mathcal{D}\}$, value functions $\{V, W\}$ and a price $q(B_1^D)$ such that

i Given the price, $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{B}_1\}$ solves the domestic investor's problem at every state, and W attains the maximum;

ii $q(B_1^D)$ satisfies the break-even condition of foreign investors;

iii $\mathcal{D}$ solves the government problem at every state, and V attains the maximum;

iv Given the price function, the policy of domestic investors $\mathcal{B}_1$ is consistent with the aggregate demand B_1^D;

I conclude this section by describing the sufficient conditions for a Markov equilibrium in terms of the main-interest variable, B_1^D. The following lemma characterizes the Markov equilibrium.

Lemma 1. *(Sufficient Conditions for Markov Equilibrium) Given an initial debt B_0, and government borrowing B_1; consider a level of domestic debt x such that*

(i) $x \in [0, B_1]$, *and* $B_0 = \left(1 + \beta[1 - F(\bar{V}(x))]\right)(B_1 - x)$ *or;*

(ii) $x = 0$, *and* $B_0 > \left(1 + \beta[1 - F(\bar{V}(x))]\right)(B_1 - x)$.

Then, it is possible to construct all functions of a Markov equilibrium by making $x = B_1^D$.

Proof. The proof is given in Appendix A. □

The lemma 1 summarizes all the conditions of a Markov equilibrium for two cases: when domestic investors are borrowing constrained and where they are not. By (18) and the strict concavity of $u(c)$, we know that when domestic investors are not borrowing constrained, there is perfect consumption smoothing between the first period and states where the government repays. Thus, the first condition on Lemma 1 states that for any positive value of B_1^D consistent with a Markov equilibrium, consumption in the first and second periods should be equal under repayment. Furthermore, the second condition of Lemma 1 is the equilibrium condition when domestic investors are restricted in borrowing. In such cases, $B_1^D = 0$, and consumption in the first period exceeds consumption in the second period under repayment.

4 Multiplicity

In this section, I study the properties of the Markov equilibrium for different parametrizations. We begin by providing a graphical illustration of an environment that features multiplicity. Next, I will derive an analytical condition that leads to the existence of multiple Markov equilibria.

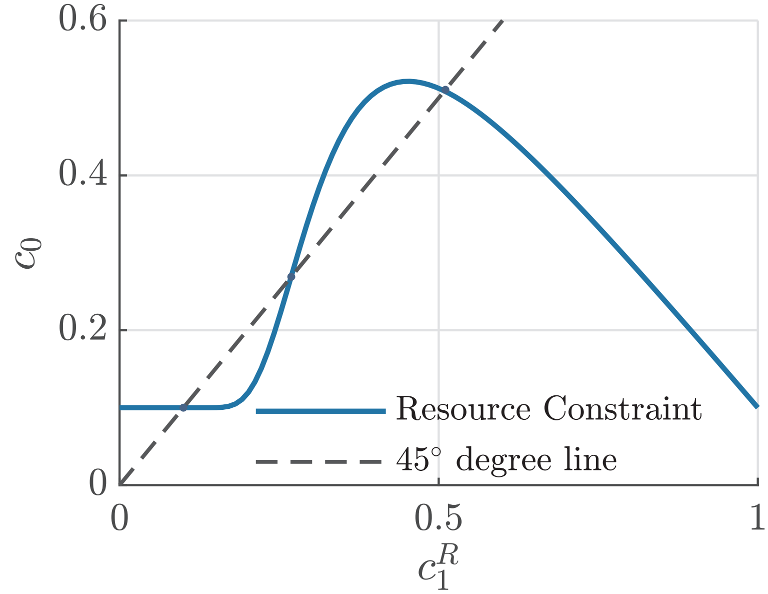
4.1 Graphical Illustration.

Consider an economy with an initial level of debt $B_0 = 0.9$, where the government issues $B_1 = 1$. In Figure 1, I use (20) and (21) to compare consumption under repayment in the first and second periods. The solid blue line represents all the combinations of consumption that are consistent with the resource constraint of the economy. The dashed black line represents the 45° degree line. As established in Lemma 1, the equilibrium condition coincides with the 45° degree line, so at any equilibrium, consumption in the first period and in the second period under repayment should be the same. From the plot, we can see that in this economy, there exist three different Markov equilibria.

To understand the intuition behind this result, let us analyze the resource constraint of this economy. We begin in the corner where domestic investors allocate the maximum possible amount of resources to consumption in the second period. At this point, all borrowing becomes domestic debt ($B_1^D = B_1$). Since the government is benevolent and there is a strictly positive cost of default, the probability of default is zero and c_1^R is very high. For levels of c_1^R close to this point, the probability of default is low and the slope of the boundary of the resource constraint is β. In this region, the trade-off between consumption in the first and second periods resembles the textbook consumption-saving problem, where domestic investors use government debt to carry consumption from the first to the second period at a price β.

However, as we reduce c_1^R, the probability of default increases. This leads to a point where the economy reaches the endogenous limit of foreign borrowing. In this second region, consumption in both periods decreases as domestic savings decrease. Consumption in the second period decreases because domestic investors' savings are decreasing. Specifically, as foreign debt increases, the probability of default increases, decreasing the prices of government bonds. Consequently, the value of government bonds held by foreign investors decreases, reducing resources in the economy and consumption in the first period.

Figure 1: Consumption



Note: In this figure, I use a truncated normal distribution for ν, where $\mu_\nu = 2$ and $\sigma_\nu = 1$, $B_0 = 0.9$, $B_1 = 1$ and $y = 1$.

As we continue to reduce c_1^R, we reach the point where government bonds have no value because the probability of default is close to one. In this region, the resource constraint is a horizontal line because neither the government nor domestic investors can trade consumption from the first to the second period.

There are three key ingredients for the multiple equilibria. The first is the non-monotonic shape of the resource constraint of the economy, which is explained by the effect of domestic demand for government bonds on prices. The second is that the domestic demand for government bonds (B_1^D) depends on the level of taxes (or transfers) of the government, which in turn depends on the value of government bonds in the first period. The third one is that the government cannot control the level of foreign debt, and domestic investors do not internalize the effect of their demand on the government's incentives to default. The combination of those three factors implies that the Markov equilibrium conditions could be satisfied in the wrong part of the resource constraint of the economy, where it is possible to increase consumption in both periods by increasing domestic demand for bonds.

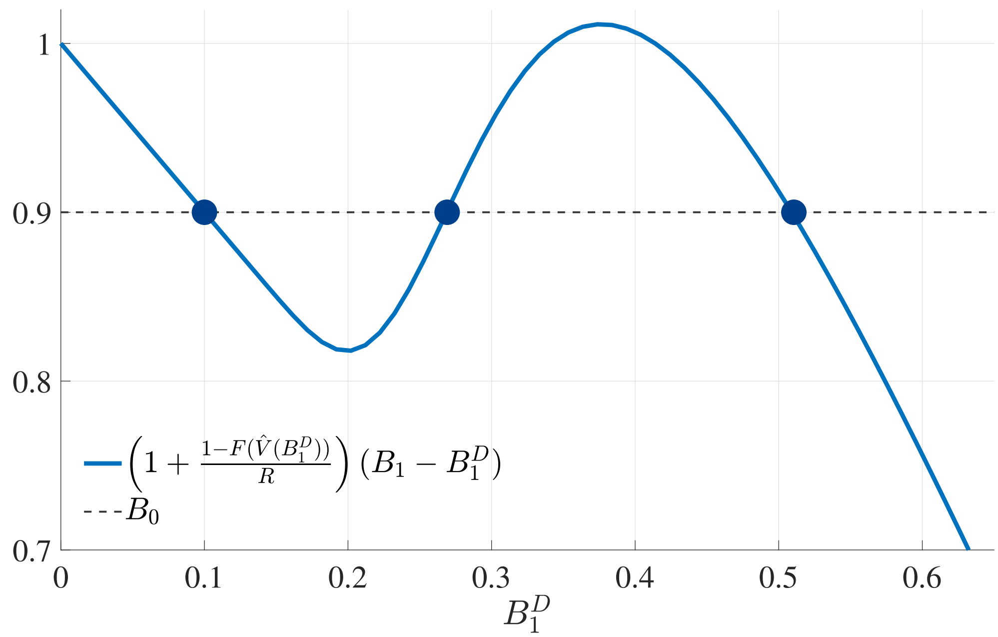
These forces could lead the economy to multiple equilibria as follows. Suppose that investors expect high domestic demand for government bonds (B_1^D). By equation (13), they expect a lower probability of default, and the price of government bonds is high. High prices of government bonds imply that the government sets low taxes (or high transfers), relaxing the budget constraint of the domestic investors. Consequently, domestic investors choose a high B_1^D to equate the marginal utility of consumption in the first and second periods under repayment, which ultimately confirms the optimistic expectation. However, note that the nonmonotonic shape of the resource constraint creates the possibility of another two equilibria. Starting with optimistic equilibrium, if domestic investors reduce their demand for government bonds, consumption in the second period under repayment goes down because the savings of domestic investors are decreasing. Moreover, consumption in the first period also decreases due to the effect of higher foreign debt on prices. The result is that the marginal utility of consumption in both periods increases, and the Euler equation has two other roots, where consumption in both periods is lower.

Take away. The resource constant of the economy resembles the form of a debt Laffer curve, where, for a given borrowing of the government, the value of foreign debt has an endogenous maximum. Because the government cannot control the composition of the bondholders, the decentralized economy could lie on the wrong side of the curve, where it is feasible to reduce foreign debt and increase the economy's resources in the first period. Domestic investors do not internalize the effect of aggregate domestic demand on the government's incentives to repay, and, in turn, the effect on bond prices. Because there are strategic commentaries between domestic investors, this inefficiency is boosted at the aggregate level and could lead the economy to multiple equilibria.

4.2 Conditions for Multiple Equilibria

The purpose of this subsection is to analyze the economic conditions that enable the existence of multiple equilibria. As I emphasized in the previous analysis, what is crucial for multiple equilibria is the nonmonotonic shape of the resource constraints of the economy. Let us return to the analysis of equilibrium conditions. In Figure 2 I plot condition (i) of Lemma 1. The black dotted line is the initial condition B_0, and the solid blue line is the part of the equilibrium condition that is a function of B_1^D. Consistent with Figure 1, both lines intersect three times, so the economy has three Markov equilibria.

Figure 2: Equilibrium Condition



Note: In this figure, I use a truncated normal distribution for ν, where $\mu_\nu = 2$ and $\sigma_\nu = 1$, $B_0 = 0.9$, $B_1 = 1$ and $y = 1$.

It is easy to check that the derivative of the equilibrium condition at $B_1^D = 0$ and $B_1^D = B_1$ is negative. Then, there are multiple equilibria if an intermediate region exists where the equilibrium condition increases in B_1^D. In the next proposition, I formalize the argument. This proposition establishes the main result of the paper.

Proposition 1. *(Multiple Equilibria) Assume for some level of foreign debt* $w \geq 0$

$$u'(y-w)f(u(y)-u(y-w))w - [1-F(u(y)-u(y-w))]\beta > 1$$

Then, there exists a sequence of debt $\{B_0, B_1\}$*, such that there are multiple Markov equilibria.*

Proof. The proof is given in Appendix B. □

Proposition 1 establishes conditions of the environment that lead to the existence of multiple equilibria. In particular, it sets conditions on the government's preferences and the distribution of the default cost such that for some value of foreign debt, the price elasticity with respect to the foreign debt is sufficiently high to support multiple equilibria. The intuition of this result is that at least for one level of foreign debt, the effect of domestic demand for bonds should be high enough so that when domestic investors reduce their demand for bonds, consumption in the first period goes down faster than consumption in the second period at states of repayment. If such a point exists, it is possible to construct an economy with multiple equilibria.

In Appendix B I show that multiple equilibria emerge only when the government issues a sufficiently large B_1. This result can be intuitively understood as follows. When the government issues a small amount of debt, the risk of default is low. Consequently, the price elasticity with respect to foreign debt $B_1 - B_1^D$ remains relatively low at different values of B_1^D. In such cases, the price of debt is primarily determined by the total borrowing, resembling the behavior of Eaton-Gersovitz's model with a unique equilibrium.

Finally, multiple equilibria emerge only for intermediate levels of initial debt B_0. We can see in Figure 2 that for low levels of B_0 there exists only one equilibrium. In Appendix B I formalize this condition. The intuition behind this result is related to the feedback loop between bond prices and the domestic demand for government bonds. When the initial level of debt is high, the resource constraint is very tight, and increases in taxes in response to low bond prices hike the marginal utility of domestic investors. These hikes push the model toward a unique equilibrium characterized by a low share of domestic debt and low prices of government bonds. Conversely, with low initial debt, fiscal adjustments to low bond prices have a mild effect on the marginal utility of consumption, resulting in a single equilibrium marked by high domestic debt and high prices.

5 Discussion.

Before presenting the policy implications of multiplicity, I discuss how the result relates to the literature on self-fulfilling crises. In addition, I explore the empirical relevance of the mechanisms of the model and suggest a possible avenue to test them in empirical work.

5.1 Relation to the Literature.

First, I will compare the results of the paper with other sources of multiplicity in the literature. Second, I will discuss why the proofs of unique equilibria in sovereign default models cannot be extended to this environment.

In Calvo (1988), multiplicity arises when the government cannot adjust taxes due to low bond prices. He assumes that the government has a fixed surplus that should be financed with borrowing. In that world, pessimistic expectations about the risk of default reduce government bonds' prices, forcing the government to increase borrowing to keep constant total revenue, increasing risk, and confirming initial expectations.

Instead, in my model, I assume that the government can freely adjust taxes in response to low bond prices due to pessimistic expectations about the future risk of default. The key insight from the theory is that by increasing taxes in response to low bond prices, the government reduces the domestic demand for government bonds, increasing risk and confirming initial pessimistic expectations.

Now, let us compare the main result of the paper with other types of multiplicity. Broner et al. (2014) have a related result, which shows a case of multiplicity in a market populated by domestic and foreign investors. The driver of multiplicity in their model is that optimistic expectations about the probability of default make foreign investors increase their demand for government bonds. In their model, it implies lower domestic holdings of government debt and a lower crowding-out effect on investment. In turn, a higher investment increases the cost of default, which confirms optimistic expectations that the probability of default is low. In sharp contrast to my results, higher foreign debt decreases the probability of default in their model, owing to a lower crowding-out effect on investment.

I see their mechanism as complementary to the one in my model. The domestic demand for government bonds might have two opposite effects on the government's incentives to repay. On the one hand, it reduces foreign debt and in that way reduces the surplus from defaulting. Consequently, it reduces the government's incentive to default. However, it can crowd out investment and increase government incentives to default because the associated productivity cost is lower. The quantitative analysis of this trade-off and its implication for multiplicity is still an open question for future work.

Consider the model where only foreign investors participate in the sovereign debt market. Auclert and Rognlie (2016) argue that when the government commits to repay all its debt before issuing new bonds, the risk of default is entirely determined by the new borrowing (B_1). In addition, they show that for a given level of borrowing (B_1), there exists a unique threshold for the default cost, below which the government would always choose to repay. If this threshold is unique, the probability of repayment is unique, leading to a single equilibrium.

In contrast, in my model, there exists an intra-period risk because the fiscal policy depends on the bond prices. This in turn depends on the domestic demand for bonds. In addition, fiscal policy affects both the domestic demand for bonds and the risk of default. It implies that the threshold, which is a function of domestic demand – namely, $\bar{V}(B_1^D)$– is not unique.

Aguiar and Amador (2019) use a different argument to prove the uniqueness. First, they establish a dual problem that delivers the same allocations as the standard government maximization problem. In particular, they show that maximizing the expected profits of international investors subject to incentive-compatible constraints for the government is equivalent to maximizing the utility of the government subject to the non-arbitrage condition of the foreign lenders. They also show that Blackwell's conditions apply to this equivalent problem, allowing them to prove the uniqueness. Note that the dual problem in my environment would imply that domestic investors internalize the effect of their actions on the government's incentives to default. Because the dual problem is not equivalent to the problem of the government in the decentralized equilibrium, the proof cannot be extended to this model.

5.2 Empirical Relevance.

The main mechanism of the paper can be summarized as follows. Given a level of borrowing, if investors expect a low domestic demand for government bonds, they anticipate a high level of foreign debt in period one. Higher foreign debt induces a higher probability of default, reducing the price of the bond. Consequently, the government has to increase taxes or reduce transfers to satisfy the fiscal budget constraint. Fiscal adjustment tightens the budget constraints of domestic investors, increasing the marginal utility of consumption in the first period, reducing their demand for government bonds, and confirming pessimistic beliefs.

This mechanism implies at least three empirical implications. The first implication is that the government cannot control the foreign debt position of the economy. In this paper, I assume that sovereign debt markets are not segmented, so given the government's debt policy, the amount of foreign debt is an equilibrium outcome rather than the government's choice as in Eaton and Gersovitz (1981). An empirical validation of this assumption is that, as shown by Arslanalp and Tsuda (2014), domestic and foreign bondholders are mixed when decomposing government debt into local and foreign currency, suggesting that markets are not completely separated.

Second, the government adjusts fiscal policy in response to bond prices. This result aligns with Bianchi et al. (2023) among others, which provides empirical evidence that governments adjust fiscal deficits in response to increases in the spreads of sovereign bonds.

The third implication is that households reduce their demand for government bonds in response to fiscal adjustment. This mechanism is related to the well-studied literature on Ricardian equivalence[2]. According to this view, domestic investors adjust their savings in response to changes in fiscal policy. If the government reduces taxes (or increases transfers) by issuing debt, domestic investors anticipate higher future taxes and increase savings. Ricciuti (2003) surveys the empirical literature on Ricardian equivalence and shows that most empirical studies find evidence of an increase in savings in the economy in response to both tax reductions and an increase in transfers. However, it is still an empirical open question to estimate the elasticity of the demand for government bonds to increases in domestic savings.

Another key challenge to assess the quantitative relevance of the mechanism is to estimate the elasticity of government prices to the domestic demand for government bonds. The challenge is to determine whether there are any factors that could increase the risk of default and reduce the domestic demand for bonds in the data. This problem makes it difficult to estimate the effect of increased domestic demand for government bonds on the price. I leave those questions open for future work.

6 Normative Analysis

In this section, I will analyze the efficiency properties of the equilibria. First, I provide a graphical illustration to show that domestic demand is too low compared to the efficient level and how the size of the inefficiency critically depends on investors' expectations. Second, I formalize the result.

6.1 Constrained Efficiency.

Consider the problem of a social planner who directly chooses the level of foreign debt $B_1 - B_1^D$, as in Eaton-Gersovitz. Taking the policy function of the government in the second period as given and the price function, she solves the following.

$$V = \max_{B_1^D} u(c_0) + \beta\mathbb{E}[u(c_1(\nu, B_1^D)) - d(\nu, B_1^D)\nu], \tag{22}$$

[2]Recent surveys by Ricciuti (2003) and Ljungqvist and Sargent (2018) provide comprehensive overviews of this literature.

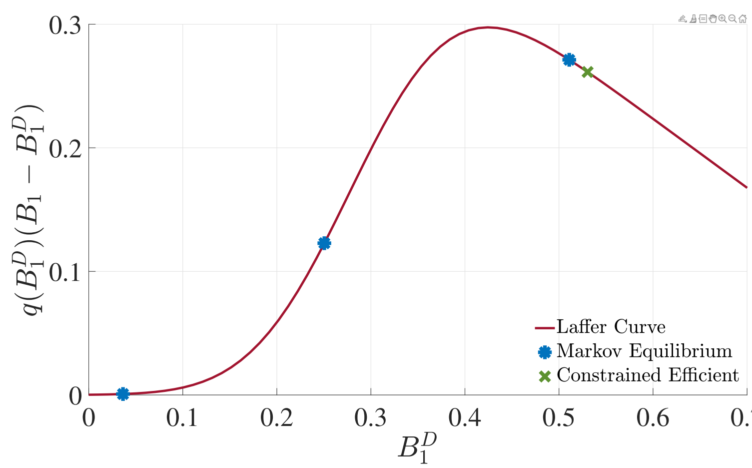
subject to

$$
\begin{aligned}
c_0 + q(B_1^D)(B_1 - B_1^D) = & \; y - B_0 \\
c_1(\nu, B_1^D) + (1 - d(\nu, B_1^D))(B_1 - B_1^D) = & \; y.
\end{aligned}
$$

where the price function $q(B_1^D)$ resembles Eaton-Gersovitz, where the planner internalizes that q is a function of $B_1 - B_1^D$.

Figure 3 illustrates the value of foreign debt as a function of the aggregate domestic demand for bonds.[3] The green "x" is the efficient constrained level of domestic debt. The blue circles are domestic debt levels consistent with Markov equilibria. Note that for two of the Markov equilibria, increasing resources in the first period and reducing foreign debt in the second period by increasing domestic demand for government bonds are possible. Also, in one of the Markov equilibria, the value of foreign borrowing in the first period is only 3% of the value in the efficient allocation restricted. In comparison, the face value of foreign debt in the second period is almost 50% higher than in the constrained-efficient allocation.

Figure 3: Value of Foreign Debt



Note: In this figure, I use a truncated normal distribution for ν, where $\mu_\nu = 2$ and $\sigma_\nu = 1$, $B_0 = 0.9$, $B_1 = 1$ and $y = 1$.

In general, all levels of domestic debt consistent with Markov equilibrium con-

[3] In this example, I used the same parameters as in section 4

ditions are below the constrained-efficient allocation. I formalize the result in the following proposition:

Proposition 2. *(Constrained inefficiency) The decentralized Markov Equilibrium is not constrained-efficient.*

Proof. The proof is given in Appendix C. □

Proposition 2 compares the allocations consistent with the decentralized Markov equilibrium, where domestic investors choose their demand of government bonds, with the allocations of the social planner who is constrained to the same commitment problem as the government in the decentralized equilibrium, but chooses the level of foreign debt as in Eaton-Gersovitz.

There is only one candidate of the Markov equilibrium to be constrained-efficient; the Markov equilibrium with more domestic debt. Note that for the other two equilibria, it is possible to increase consumption in both the first and second periods under repayment by increasing the domestic demand of government bonds, which makes it trivial to prove that they cannot be efficient.

Furthermore, even the best Markov equilibrium is inefficient. The intuition of the result is as follows. Domestic investors fail to internalize that increasing the domestic demand for bonds reduces the government's incentives to default in the second period, which increases prices and relaxes the resource constraint. Then, in general, the optimality conditions of the social planner and the domestic investors in the decentralized economy do not coincide.

The result is related to the literature on the composition of sovereign debt; see, for example, Chari et al. (2020) and Perez (2018), which find that the domestic demand for government bonds is too low compared to the solution of a social planner who internalized the effect of the aggregate demand for bonds.

In Perez (2018) and Broner et al. (2014), the assumption that the government cannot selectively default on foreign investors is the key to delivering a suboptimal demand for government bonds. The main mechanism in those models relies on modeling a financial sector that invests in government bonds and capital. In that environment, when the government defaults, it reduces the wealth of banks, distorting investment.

Increase the cost of default for the government and bring some discipline by reducing incentives to default.

In contrast, in my model, what is important is that for a given level of debt, a lower domestic demand for bonds implies a higher level of foreign debt and an associated high probability of default. One critical implication of this different mechanism is that in my model, debt is inefficiently low even when the government can selectively default on foreign investors. This result is related to Chari et al. (2020), who describe a model in which the demand of domestic banks for government bonds is inefficiently low in a closed economy model where the government can selectively default on consumers.

Importantly, the existence of multiple equilibria does not depend on the government's ability to default only on foreign investors. Even if the government has the option to selectively default, when the government cannot control the domestic demand for bonds, there are multiple equilibria. In Section 8, I relax the assumption that the government cannot selectively default on foreign investors and discuss how the main results of the paper change in that environment.

7 Implementation.

In this section, I consider a subsidy for domestic investors' purchases of government bonds. I follow Bassetto (2005) to discuss that the government requires a subsidy contingent on all possible realizations of the domestic demand for bonds (B_1^D) to implement constrained-efficient allocation when there are multiple equilibria. Having access to such a policy helps the government choose the best equilibrium. I also explore the limitations of simpler policies in addressing the vulnerability to self-fulfilling crises.

7.1 Regulated Economy.

Government. The government has access to lump sum taxes to finance the subsidy. So, the budget constraint of the government in the first period is

$$B_0 + \tau(B_1^D) q B_1^D = T_0(B_1^D) + q B_1. \tag{23}$$

In Appendix C, I formally define the rest of the environment, which is very similar to the baseline model. We then define the competitive equilibrium of an economy with financial regulation.

Definition 3. (Competitive Equilibrium with Financial Regulation) Given a sequence of debt $\{B_0, B_1\}$ and government policies $\{\{T_0(B_1^D), \tau(B_1^D)\}_{B_1^D}, \{T_1(\nu), d(\nu)\}_\nu\}$; the equilibrium consists of a price q and allocations of domestic investors $\{c_0, b_1, \{c_1(\nu)\}_\nu\}$ such that

i given prices and government policies, $\{c_0, b_1, \{c_1(\nu)\}_\nu\}$ solves domestic investor's problem;

ii given prices and government policies, q solves (5);

iii given prices, and domestic investors allocation, $\{\{T_0(B_1^D), \tau(B_1^D)\}_{B_1^D}, \{T_1(\nu), d(\nu)\}_\nu\}$ is consistent with government budget constraints (6) and (23);

iv market clears: $B_1^D = b_1$.

7.2 Optimal Financial Regulation.

Next, I will analyze the government's optimal policy and establish conditions under which a subsidy implements the constrained-efficient allocations.

The problem of the government in the second period and the problem of the international investors are the same as in the baseline model.

Domestic Investors. On the other hand, the domestic investors' problem in this case would be

$$W(B_1^D) = \max_{c, b_1} u(c_0) + \beta \mathbb{E}[u(c_1(\nu, B_1^D)) - d(\nu, B_1^D)\nu], \tag{24}$$

subject to

$$c_0 + (1 - \tau(B_1^D))qb_1 + T_0(B_1^D) = y \qquad (\lambda_1)$$
$$c_1(\nu, B_1^D) + T_1(\nu, B_1^D) = y + (1 - d(\nu, B_1^D))b_1 \qquad (\lambda_2(B_1^D))$$
$$b_1 \geq 0. \qquad (\mu)$$

Finally, I define the Markov Equilibrium of this economy.

Definition 4. (Markov Equilibrium with Financial Regulation) Given a level of debt B_0 and borrowing B_1; a Markov perfect equilibrium is defined by a set of strategies $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{B}_1, \mathcal{D}, \mathcal{T}\}$, value functions $\{V, W\}$ and a price q such that

i given the price, $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{B}_1\}$ solves the domestic investors problem at every state, and W attains the maximum;

ii q satisfies the break-even condition of international investors;

iii given the price, $\{\mathcal{D}, \mathcal{T}\}$ solves the government problem at every state, and V attains the maximum;

Proceeding analogously to the baseline model, I first describe equilibrium conditions when domestic investors are not borrowing-constrained. It is possible to establish that, under the homotheticity of the utility function, equilibrium conditions imply that consumption in the second period under repayment is a share of consumption in the first period that depends on the subsidy. On the other hand, in states where domestic investors are borrowing constrained, $B_1^D = 0$, and consumption in the first period is greater than a certain share of consumption in repayment in the second period. Formally:[4]

Lemma 2. *(Sufficient Conditions for Equilibrium with Financial Regulation) Given the initial conditions* B_0*, the amount of borrowing* B_1*, and a subsidy* $\tau(B_1^D)$ *consider an* z *such that*

(i) $z \in [0, B_1]$*, and* $(y + \beta[1 - F(\bar{V}(z))](B_1 - z) - B_0) - [u']^{-1}[1 - \tau(z)](y - B_1 + z) = 0.$

[4]Let $[u]^{1-}$ is the inverse of the derivative of the utility function of domestic investors.

(ii) $z = 0$, *and* $(y + \beta[1 - F(\bar{V}(z))](B_1 - z) - B_0) - [u]^{-1}[1 - \tau(z)](y - B_1 + z)) < 0$;

Then, it is possible to construct all functions of a Markov equilibrium by making $z = B_1^D$.

Proof. The proof is given in Appendix C. □

7.3 Efficiency of Financial Regulation.

The previous literature studies optimal regulation in the form of a subsidy that depends only on the state of the economy. In my model, it would imply a fixed policy. The next proposition establishes that such a policy cannot guarantee the implementation of the constrained-efficient allocation.

Proposition 3. *(State Contingent Subsidy on debt) The Markov equilibrium of a regulated economy with a state-contingent subsidy on government bonds is not, in general, constrained-efficient.*

Proof. The proof is given in Appendix E. □

The proposition 3 underscores one of the primary insights derived from the model. When there are multiple equilibria, the government becomes susceptible to self-fulfilling crises. In this context, financial regulation in the form of state-contingent policies discussed in the previous literature cannot implement constrained-efficient allocations as they cannot prevent self-fulfilling crises.

Achieving efficiency would require the government to announce a sequence of subsidies for all possible realizations of domestic demand for government bonds. Formally

Proposition 4. *(Optimal Subsidy on Debt) constrained-efficient allocations can be implemented with an appropriate subsidy on debt contingent on the domestic demand for government bonds, with revenue collected with lump-sum taxes.*

Proof. The proof is given in Appendix F. □

The intuition of the result is that the government needs to implement a policy that not only corrects the inefficiency, but also aligns the expectation of the market toward the constrained-efficient allocation. To do so, the government can, for example, announce a tax on domestic holdings of debt whenever the aggregate domestic demand for bonds is inconsistent with the constrained-efficient allocation. Such a policy would help the government signal that the only possible equilibrium is the constrained-efficient one.

Chari et al. (2020) and Perez (2018) also consider a policy establishing a minimum requirement for domestic holdings of government bonds. Given that the government has access to lump-sum taxes. The public announcement of a minimum requirement for domestic holdings of bonds would be equivalent to a sequence of subsidies contingent on domestic demand (B_1^D). An important implication of multiplicity is that, in contrast to previous analysis in the literature, the announcement of the minimum requirement is key to aligning expectations and restoring efficiency in the market.

8 Extensions.

In this section, I discuss two extensions to the baseline model, which relax two sensible assumptions: first, that the borrowing of the government is exogenous and second, that the government cannot selectively default on foreign investors.

8.1 Endogenous Borrowing.

In the baseline model, I assume that the government's borrowing is exogenous. I focus on characterizing the equilibrium for any possible B_1 and derive conditions of B_1 such that there are multiple equilibria. One of the main results of the paper is that the government is vulnerable to facing self-fulfilling crises when borrowing is high. I leave the analysis of the optimal debt policy for future work. However, in Appendix G, I describe the problem of the government and discuss conditions under which the government might choose to issue a level of debt in the multiplicity region.

In Appendix G, I define the problem of investors and government in period one, with borrowing being a state rather than a parameter. Also, a sunspot is used to select the equilibria in the case of multiplicity. Next, I extend the model defining the problem of the government in the first period. Finally, I discuss how the government might choose to issue a high level of borrowing B_1, which makes it vulnerable to facing self-fulfilling crises when the initial stock of debt is high or when the probability of the sunspot selecting the equilibria with low domestic demand for bonds is relatively low.

8.2 Non-Selective default.

In the baseline model, domestic demand for government bonds is inefficiently low. In contrast with the literature on the composition of sovereign debt bondholders, the assumption that the government cannot selectively default on foreign investors is not essential for the result, and consequently, it is not essential for multiplicity. To illustrate this point, in Appendix H, the assumption is relaxed that the government cannot selectively default on foreign investors. I define the equilibrium, solve the model, and show a numerical example of multiplicity.

The mechanism operates as follows. Given a level of borrowing, when the aggregate domestic demand for bonds is low, foreign investors hold a high level of debt. Consequently, the price of foreign bonds decreases. The government collects less revenue and is forced to increase taxes. Domestic investors must pay higher taxes (or reduce transfers), so the domestic demand for government bonds decreases. Just as in the baseline model, an economy where the government can selectively default is vulnerable to self-fulfilling crises due to the feedback loop between the domestic demand for bonds and the resource constraint of the economy.

This result effectively extended the results of Chari et al. (2020) who argue that when the government cannot selectively default, domestic debt brings discipline and increases prices. Instead, in this model, high domestic demand for government bonds reduces risk because a given government debt policy reduces the amount of foreign debt, reducing the risk of default and increasing prices. The key insight is that, in this model, there is space for policy interventions in the domestic market regardless of whether the government can or cannot selectively default on foreign debt.

9 Conclusion

This paper shows a sovereign debt model with domestic and foreign investors that supports multiple equilibria. In particular, I show conditions on the elasticity of the price of the bonds to the foreign debt that support multiplicity in the model.

The main force driving multiplicity is the strategic complementarities between domestic investors. When domestic demand for bonds is high, the risk of default is low and the resource constraint is relaxed because the government can borrow at better terms. Increasing savings and investing in government bonds is optimal for individual domestic investors. These complementarities could lead the economy to multiple equilibria.

The existence of multiple equilibria provides a theory of financial regulation. I discuss the policy implications of multiplicity and study how financial regulation could be useful in turbulent times to prevent self-fulfilling crises. I also establish that the government would need to announce a sequence of policies contingent on the domestic demand for government bonds to select the equilibrium with the best outcome.

Most of the sovereign default literature focuses on models, implicitly assuming that the government has enough instruments to choose the constrained, efficient level of foreign debt. One of the models' lessons is that, in the presence of multiple equilibria, such instruments are a complex sequence of subsidies.

This paper leaves several open questions for future research. One of the most important might be the role of the composition of bondholders in models with long-term debt and how it interacts with the well-studied inefficiencies of that type of debt.

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A Proof: Lemma 1

The proof is by construction. We use (18) and replace all equilibrium conditions: (2), (4), (6), (7), (13), (42), and (11) to get the equilibrium conditions are

(i) $B_1^D = 0$ and $u'(y + \beta[1 - F(\bar{V}(B_1^D))](B_1 - B_1^D) - B_0) - u'(y - B_1 + B_1^D) > 0$;

(ii) $B_1^D \in [B_1]$ and $u'(y + \beta[1 - F(\bar{V}(B_1^D))](B_1 - B_1^D) - B_0) - u'(y - B_1 + B_1^D) = 0$.

Note that the strict concavity of $u(.)$ implies the result.

B Proof: Proposition 1

B.1 Step 1: Continuous Functions.

Let us define consumption as a function of the composition of domestic debt (B_1^D):

$$c_0(B_1^D) = y + \beta[1 - F(\bar{V}(B_1^D))](B_1 - B_1^D) - B_0 \tag{25}$$

$$c_1^R(B_1^D) = y - B_1 + B_1^D. \tag{26}$$

Note that consumption functions are continuous for the relevant space of B_1.

B.2 Step 2: Existence of equilibrium

From Lemma 1, the equilibrium conditions are as follows.

$$B_0 = (1 + \beta[1 - F(\bar{V}(B_1^D))])(B_1 - B_1^D). \tag{27}$$

Note that, since consumption is a continuous function on domestic debt, $(1+\beta[1-F(\bar{V}(B_1^D))])(B_1 - B_1^D)$ is a continuous function on B_1^D, so there exists a B_0 such that (27) holds.

B.3 Step 3: Multiple Equilibria.

By condition (i), there exists an w such that:

$$u'(y-w)f(u(y)-u(y-w))w-[1-F(u(y)-u(y-w))]\beta>1. \tag{28}$$

Let B_0 be such that:

$$B_0=(1+\beta[1-F(\bar{V}(w))])(B_1-w). \tag{29}$$

And let $B_1>w$. Then we can make $B_1^D=B_1-w$. Such a level of domestic debt would be an equilibrium. Also, the derivative of the equilibrium condition at that level of domestic debt is negative. Because this is an economy with two goods, we can apply the index theorem in Dierker (1972) to establish there exist at least three equilibria in this economy. See Kehoe (1991) for a discussion of the index theorem.

C Proof: Proposition 2

First, recall the problem of the social planner:

$$V=\max_{B_1^D} u(c_0)+\beta\mathbb{E}[u(c_1(\nu,B_1^D))-d(\nu,B_1^D)\nu], \tag{30}$$

subject to

$$\begin{aligned} c_0+q(B_1^D)(B_1-B_1^D) &= y-B_0 \\ c_1(\nu,B_1^D)+(1-d(\nu,B_1^D))(B_1-B_1^D) &= y. \end{aligned}$$

Then, the FOC with respect to B_1^D is:

$$u'(c_0)(u'(y - B_1^D)f(u(y) - u(y - B_1^D))(B_1 - B_1^D) - [1 - F(u(y) - u(B_1^D))]\beta) \\ - \beta[1 - F(u(y) - u(B_1^D))]u'(c_1^R) = 0 \quad (31)$$

Also define, the domestic debt that maximizes the value of foreign government debt as:

$$B_1^{D*} = \underset{B_1^D}{\text{argmax}}\{\beta[1 - F(u(y) - u(B_1^D))](B_1 - B_1^D)\}$$

Note for any $B_1^D < B_1^{D*}$ all the terms in 31 are negative, so the FOC never applies. Then, we can rewrite the problem of the planner as follows:

$$V = \max_{B_1^D} u(c_0) + \beta\mathbb{E}[u(c_1(\nu, B_1^D)) - d(\nu, B_1^D)\nu], \quad (32)$$

subject to

$$\begin{aligned} c_0 + q(B_1^D)(B_1 - B_1^D) = &\ y - B_0 \\ c_1(\nu, B_1^D) + (1 - d(\nu, B_1^D))(B_1 - B_1^D) = &\ y. \\ B_1^D < &\ B_1^{D*} \end{aligned}$$

Due to the concavity of u, the objective function of this equivalent problem is strictly concave. Also, the set of constraints of this problem is convex. Then, FCO is necessary and sufficient conditions to characterize the problem's solution.

The last part of the proof is in contradiction. Assume that there exists a level of domestic debt consistent with the Marvov equilibrium condition in Lemma 1 and the FOC of the planner problem. Then, at that value of domestic debt $c_1^R = c_0$. This implies:

$$u'(y - B_1^D) f(u(y) - u(y - B_1^D))(B_1 - B_1^D) = 0$$

Then, it could be $B_1 = B_1^D$, which contradicts $c_1^R = c_0$ for any $y - B_0 \neq 0$. It could also be $0 = B_1^D$, which also contradicts $c_1^R = c_0$ for $B_1 > 0$

D Economy with Subsidy on Government Bonds

In this appendix, I describe the economy in which the government has access to an additional policy tool, a subsidy for domestic investors on government bonds.

D.1 Domestic Investors.

Representative domestic investors have time-separable preferences with utility over consumption $\{c_t\}$ represented by

$$U = u(c_0) + \beta \mathbb{E}[u(c_1(\nu)) - d(\nu)\nu]. \tag{33}$$

The budget constraint of domestic investors in the first period is

$$c_0 + q(1 - \tau(B_1^D))b_1 = T_0 \tag{34}$$

$$b_1 \geq 0. \tag{35}$$

In period one at the state s, the budget constraint of the domestic investors becomes:

$$c_1(\nu) + T_1(\nu) = y + b_1(1 - d(\nu)). \tag{36}$$

The domestic investors' problem is to choose the sequences $\{c_0, c_1(\nu), b_1\}$ to maximize (33) subject to (34), (35) and (36).

D.2 International Lenders.

The international lenders have the same problem as in the baseline model.

D.3 Government.

The fiscal budget for the first period is

$$B_0 + \tau(B_1^D)qB_1^D = T_0(B_1^D) + qB_1. \tag{37}$$

In period one at each ν, the fiscal budget becomes

$$(1 - d(\nu))B_1 = T_1(\nu). \tag{38}$$

D.4 Aggregate Resource Constraint.

We can add the budget constraints of the representative household and the government to obtain the resource constraints of the economy.

$$c_0 + B_0 = y + q(B_1 - B_1^D) \tag{39}$$

$$c_1(\nu) = y - (1 - d(\nu))(B_1 - B_1^D). \tag{40}$$

E Proof: Proposition 3

Let $\hat{B_1^D}$ be the constrained-efficient domestic debt that solves (22). By derivations of Proposition 2, we can deduce the solution of the social planner's problem is unique. Then, combing 2 and 31, the subsidy that can implement constrained-efficient allocations should be:

$$\tau = 1 - \frac{u'(y - \hat{B_1^D}) f(u(y) - u(y - \hat{B_1^D}))(B_1 - \hat{B_1^D}) - [1 - F(u(y) - u(\hat{B_1^D})]\beta)}{[1 - F(u(y) - u(\hat{B_1^D}))]\beta} \tag{41}$$

Proceeding analogously than Proposition 1, assume there exists a level of domestic debt h such that:

$$u'(y - B_1 + h) f(u(y) - u(y - B_1 + h))(B_1 - h) - [1 - F(u(y) - u(y - B_1 + h))]\beta >$$
$$[u']^{-1} \left(1 - \frac{u'(y - \hat{B_1^D}) f(u(y) - u(y - \hat{B_1^D}))(B_1 - \hat{B_1^D}) - [1 - F(u(y) - u(\hat{B_1^D}))]\beta}{[1 - F(u(y) - u(\hat{B_1^D}))]\beta} \right)$$

Then, there exist multiple equilibria in the regulated economy. Note that given the uniqueness of the social planner's problem, at most, one of them can be efficient, which implies the result.

F Proof: Proposition 4

Let $\hat{B_1^D}$ be the constrained-efficient function that solves (22). Also, let B_1^{D*} be defined as:

$$B_1^{D*} = \underset{B_1^D}{\operatorname{argmax}} \{\beta[1 - F(u(y) - u(B_1^D))](B_1 - B_1^D)\}$$

Then define

$$
\tau(B_1^D) = \begin{cases} -\gamma \quad \text{if} \quad B_1^D < B_1^{D*}, \\ 1 - \frac{u'(y-B_1^D)f(u(y)-u(y-B_1^D))(B_1-B_1^D)-[1-F(u(y)-u(B_1^D))]\beta}{[1-F(u(y)-u(B_1^D))]\beta} \quad \text{otherwise.} \end{cases} \tag{42}
$$

Where $\gamma > 0$ First, note domestic investors would never choose $B_1^D > \hat{B_1^D}$. Also, for $B_1^D > \hat{B_1^D}$, the resource constrains are a convex set. The FOC of the social planner is an increasing function. Then, the intersection of both lines is unique. Finally, by construction, the intersection coincides with the solution of (31).

G Endogenous Government Borrowing

In this appendix, I describe the problem of a government that chooses endogenously to borrow B_1. I assume the timing is as follows: at the beginning of period zero, the government chooses the level of borrowing (B_1). After the bond auction, the government observes the price and adjusts taxes to clear the fiscal budget.

G.1 Problem of the Government in period one

I solve the model by backward induction, so in this case, government borrowing (B_1) will be an additional state in the second period. In particular, the relevant state for the government becomes $\{B_1, B_1^D, \nu\}$, and the problem of the government becomes:

$$V(B_1, B_1^D, \nu) = \max_{d\in\{0,1\}} (1-d)u(y - B_1 + B_1^D) + d(u(y) - \nu). \tag{43}$$

The only choice of the government is whether to default or repay. As in the baseline model, I characterize the default decision of the government by defining the following threshold:

$$\bar{V}(B_1, B_1^D) \equiv u(y) - u(y - B_1 + B_1^D). \tag{44}$$

Given the threshold, the default function of the government is

$$d(B_1, B_1^D, \nu) = \begin{cases} 1 & \text{if} \quad \nu < \bar{V}(B_1, B_1^D), \\ 0 & \text{otherwise.} \end{cases} \tag{45}$$

Also, the probability of default is

$$F(\bar{V}(B_1, B_1^D)) = \int_{\underline{\nu}}^{\bar{V}(B_1, B_1^D)} f(\nu) d\nu. \tag{46}$$

Finally, using the fiscal budget of the government, we can solve the tax policy for the second period as follows:

$$T_1(B_1, B_1^D, \nu) = \begin{cases} 0 & \text{if} \quad \nu < \bar{V}(B_1, B_1^D), \\ B_1 & \text{otherwise.} \end{cases} \tag{47}$$

G.2 Foreign Investors.

Using the probability of default and the break-even condition of international investors, we have the following:

$$q(B_1, B_1^D) = \beta[1 - F(\bar{V}(B_1, B_1^D))]. \tag{48}$$

Combining this price function and the budget constraint of the government in the first period, the tax policy in the first period is given by:

$$T_0(B_1, B_1^D) = B_0 - \beta[1 - F(\bar{V}(B_1, B_1^D))]B_1. \tag{49}$$

G.3 Problem of the Domestic Investors.

Next, I use the government policies and the price to define the problem of domestic investors as:

$$W(B_1, B_1^D) = \max_{b_1} u(c_0) + \beta\mathbb{E}[u(c_1(B_1, B_1^D, \nu)) - d(B_1, B_1^D, \nu)\nu], \tag{50}$$

subject to

$$c_0 + q(B_1, B_1^D)b_1 + T_0(B_1, B_1^D) = y \qquad (\lambda_1)$$

$$b_1 \geq 0. \qquad (\mu)$$

$$c_1(B_1, B_1^D, \nu) + T_1(B_1, B_1^D, \nu) = y + (1 - d(B_1, B_1^D, \nu))b_1 \qquad (\lambda_2(\nu))$$

Let μ be the multiplier of the borrowing constraint. Then, the FOCs for domestic investors are

$$0 = \mu\Big(q(B_1, B_1^D)u'(c_0) - \beta[1 - F(\bar{V}(B_1, B_1^D))]u'(c_1^R)\Big) \tag{51}$$

$$b_1 = 0 \quad if \quad \mu > 0. \tag{52}$$

Note that up to this point, the model behaves very similarly to the model with exogenous borrowing. The main change is that borrowing is now a state variable in the problem of investors and government in the second period rather than a parameter.

G.4 Multiplicity and Sunspot.

In this subsection, I characterize the multiplicity region and use a sunspot to select equilibria in case of multiplicity. First, it is useful to define:

$$\mathbb{B}^{\mathbb{D}}{}_1(B_1) = \{x : x \in [0, B_1], B_0 = (1+\beta[1-F(\bar{V}(x))])(B_1-x); x = 0, B_0 > (1+\beta[1-F(\bar{V}(x))])(B_1-x)|B_1$$

Those are the values of domestic demand for bonds consistent with the Markov equilibrium for a given debt policy. Notice it is a correspondence because three values are consistent with equilibrium in the multiplicity region. Using conditions in Proposition 1 we define:

$$\overline{B_1} = \min(w) \;\; \text{s.t} \;\; u'(y-w)f(u(y)-u(y-w))w - [1 - F(u(y)-u(y-w))]\beta = 1$$

By Proposition 1, we know there are multiple solutions to the Euler equation of households if $B_1 > \overline{B_1}$. Note I am explicitly assuming that given the government's preferences and the distribution of the default cost $\overline{B_1}$ does exist.

In the case of multiplicity, there are three values of domestic demand for bonds consistent with equilibrium conditions. However, using the Index theorem, we know only two of them are stable equilibria[5]. Consequently, I use a sunspot ξ to select

[5]See Kehoe (1991) for details

between the two stable equilibria in case of multiplicity, I define the domestic demand for government bonds as:

$$b_1(B_1) = \begin{cases} \mathbb{B^D}_1(B_1) & \text{if} \quad B_1 < \overline{B_1}, \\ \max \ \mathbb{B^D}_1(B_1) & \text{if} \quad B_1 \geq \overline{B_1} \quad \text{and} \quad \xi = 1, \\ \min \ \mathbb{B^D}_1(B_1) & \text{if} \quad B_1 \geq \overline{B_1} \quad \text{and} \quad \xi = 0. \end{cases} \tag{53}$$

This demand function underscores the main result of this Appendix. If the government chooses a high level of borrowing, it will face multiplicity in the domestic demand for bonds.

G.5 Problem of the Government in period zero

We are ready to define the problem of the government in the first period. It solves:

$$V_1 = \max_{B_1} u(c_0) + \beta \mathbb{E}[u(c_1(B_1, B_1^D, \nu)) - d(B_1, B_1^D, \nu)\nu], \tag{54}$$

subject to

$$\begin{aligned} c_0 &= y + \beta[1 - F(\bar{V}(B_1, B_1^D))](B_1 - B_1^D) - B_0 \\ c_1^R &= y - B_1 + B_1^D \\ c_2^D &= y \\ B_1^D &= b_1(B_1), \end{aligned}$$

The last constraint is the implementability constraint of the government, which states that households should optimize their demand for bonds in any equilibria. Note that when B_0 is relatively high and the probability of $\xi = 1$ is low, the government might choose to enter the multiplicity region and become exposed to the risk of a self-fulfilling crisis.

H Economy without Selective Default

I describe the economy where the government can selectively default on international investors. I will assume that the government has a fixed debt policy in the first period (B_1) and conducts an auction where domestic and foreign investors can submit bids with a price and quantity for government bonds. The key friction of this model is that the government cannot control the result of the action as in the baseline model.

Because domestic investors consider that the government will not default on the debt they buy, they will offer a risk-free price (β). However, foreign investors will submit bids at prices that reflect the risk of default. For simplicity, I will assume the government implements a discriminatory price action so it would pay any investor the price it bids. Because I assume there are lump sum taxes, this assumption is made without loss of generality[6]

H.1 Domestic Investors.

The representative domestic investor has time-separable preferences with utility over consumption $\{c_t\}$ represented by

$$U = u(c_0) + \beta\mathbb{E}[u(c_1(\nu)) - d(\nu)\nu]. \tag{55}$$

The budget constraint of domestic investors in the first period is

$$c_0 + \beta b_1 = T_0 \tag{56}$$

$$b_1 \geq 0. \tag{57}$$

In period one, in each ν, the budget constraint of the domestic investors becomes:

$$c_1(\nu) + T_1(\nu) = y + b_1. \tag{58}$$

[6]This is because there is Ricardian equivalence, so the price of the domestic bonds is irrelevant.

The problem of domestic investors is to choose sequences $\{c_0, c_1(\nu), b_1\}$ to maximize (55) subject to (56), (57) and (58).

H.2 Government.

The fiscal budget for the first period is

$$B_0 = T_0(B_1^D) + q(B_1 - B_1^D) + \beta B_1^D. \tag{59}$$

In period one at each ν, the fiscal budget becomes

$$(1 - d(\nu))(B_1 - B_1^D) + B_1^D = T_1(\nu). \tag{60}$$

H.3 Aggregate Resource Constraint.

We can add the budget constraints of the representative household and the government to obtain the resource constraints of the economy.

$$c_0 + B_0 = y + q(B_1 - B_1^D) \tag{61}$$

$$c_1(\nu) = y - (1 - d(\nu))(B_1 - B_1^D). \tag{62}$$

Note the resource constraint of the economy remains the same as in the baseline model. This is key because multiplicity in the model is driven by the resource constraint which is equivalent in both models.

H.4 Competitive Equilibrium.

Now, we are ready to define the competitive equilibrium.

Definition G1. (Competitive Equilibrium) Given a debt $\{B_0, B_1\}$ and government policies $\{T_0, \{T_1(\nu), d(\nu)\}_\nu\}$, an equilibrium consists of a price q and domestic investor allocations $\{c_0, \{c_1(\nu)\}_\nu, b_1\}$ such that

i Given prices and government policies, $\{c_0, \{c_1(\nu)\}_\nu, b_1\}$ maximizes (1) subject to (56), (57) and (58);

ii Given government policies, q satisfies (5);

iii Given prices and domestic investor's allocations, $\{T_0, \{d(\nu), T_1(\nu)\}_\nu\}$ is consistent with government budget constraints (59) and (60);

iv market clears: $B_1^D = b_1$.

H.5 Optimal Policy

The problem of the government in the second period depends only on the resource constraint of the economy, which is equivalent to the baseline model. In addition, the problem of international investors remains the same because their problem depends only on the probability of default.

On the other hand, the problem of domestic investors in the first period becomes:

$$W(B_1^D) = \max_{b_1} u(c_0) + \beta \mathbb{E}[u(c_1(B_1^D, \nu)) - d(B_1^D, \nu)\nu], \tag{63}$$

subject to

$$\begin{aligned} c_0 + \beta b_1 + T_0(B_1^D) &= y && (\lambda_1) \\ b_1 &\geq 0. && (\mu) \\ c_1(B_1^D, \nu) + T_1(B_1^D, \nu) &= y + b_1 && (\lambda_2(\nu)) \end{aligned}$$

Let μ be the multiplier of the borrowing constraint. Then, the FOCs for domestic investors are

$$0 = \mu\Big(\beta u'(c_0) - \beta[1 - F(\bar{V}(B_1^D))]u'(c_1^R) - \beta[F(\bar{V}(B_1^D))]u'(c_1^D)\Big) \quad (64)$$

$$b_1 = 0 \quad if \quad \mu > 0. \quad (65)$$

Where c_1^R and c_1^D stand for consumption in the second period when the government repays and defaults, respectively. The main difference between this problem compared to the problem of the domestic investor in the base model is the fact that the government never defaults on domestic investors, so the Euler equation equates the marginal consumption in the first period to the expected marginal utility in the second period. In this environment, I define the Markov equilibrium as follows.

Definition G2. (Markov Equilibrium) Given a debt policy $\{B_0, B_1\}$, a perfect Markov equilibrium is defined by a set of strategies $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{B}_1, \mathcal{D}\}$, value functions $\{V, W\}$ and a price $q(B_1^D)$ such that

i given the price, $\{\mathcal{C}_1, \mathcal{C}_2, \mathcal{B}_1\}$ solves the domestic investor's problem at every state, and W attains the maximum;

ii $q(B_1^D)$ satisfies the break-even condition of foreign investors;

iii given the price function, $\mathcal{D}$ solves the government problem at every state, and V attains the maximum;

iv given the price function, the policy of domestic investors $\mathcal{B}_1$ is consistent with the aggregate demand B_1^D;

Same as I did with the baseline model, the sufficient conditions for a Markov equilibrium can be summarized in the following lemma:

Lemma 3. *(Sufficient Conditions for Markov Equilibrium) Given an initial condition B_0, and a government debt B_1; consider a level of domestic debt x such that*

(i) $x \in [0, B_1]$, *and* $u'(y + \beta[1 - F(\bar{V}(x))](B_1 - x) - B_0) - [1 - F(\bar{V}(x))]u'(y - B_1 + x) - [F(\bar{V}(x))]u'(y)$ *or;*

(ii) $x = 0$, *and* $u'(y + \beta[1 - F(\bar{V}(x))](B_1 - x) - B_0) > [1 - F(\bar{V}(x))]u'(y - B_1 + x) - [F(\bar{V}(x))]u'(y)$.

Then, it is possible to construct all functions of a Markov equilibrium by making $x = B_1^D$.

H.6 Multiplicity

In this subsection, I show an example of multiple equilibria in this model. The goal of this subsection is not to characterize conditions for multiple equilibria. I restrict myself to showing a case of multiplicity when there is a selective default to make clear how both multiplicity and inefficiency in the domestic bond market do not depend on the ability of the government to selectively default on domestic investors.

Figure 4: Equilibrium Condition

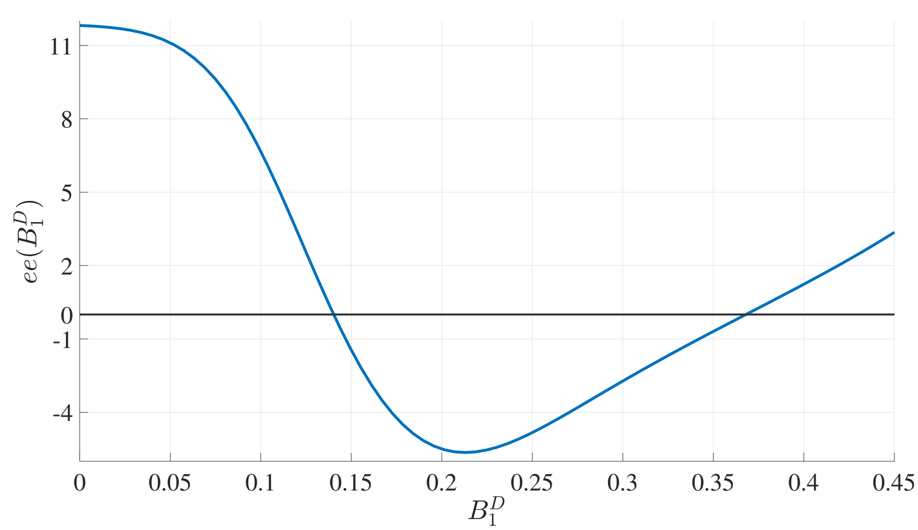

Figure 4 plots the equilibrium condition in Lemma 3. Equilibrium conditions are characterized by values of B_1^D where the blue lines cross zero or where $B_1^D = 0$ and the blue line is above zero. In this example, we can see that there are at least three equilibria. In the first equilibrium, households have borrowing constraints, and we have two other interior solutions for the Euler equation.