Debt Sustainability, Confidence Risk and International Reserves *

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Abstract

This paper analyzes how a government can use reserves to prevent a self-fulfilling crisis, as described in Lorenzoni and Werning (2019), where confidence-driven fluctuations affect bond prices. We propose a three-period model in which the government follows a fixed fiscal surplus rule and chooses the optimal reserve accumulation policy. Our analysis reveals a new mechanism for which debt-financed reserves provide insurance against self-fulfilling crises in the presence of long-term bonds. We also present empirical evidence that governments tend to accumulate reserves during periods of exceptionally high spreads and show how our theoretical framework could help explain this empirical pattern.

Keywords: Self-fulfilling debt crisis, sovereign debt, multiple equilibria, default risk, debt sustainability

JEL classification: F21,F61, E62, F64, F34, F38, F41, P41, P43.

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1 Introduction

Since the European debt crisis, a growing literature has studied the macroeconomic conditions that make economies vulnerable to confidence-driven crises. The primary concern was that rising interest rates—driven by investor anxiety—can increase future debt burdens and undermine fiscal sustainability, becoming self-fulfilling. This feedback loop, where high borrowing costs push a country toward insolvency, justifying those high rates in the first place, has heightened interest in the role of confidence in sovereign debt markets. This paper explores how foreign reserve accumulation can mitigate such dynamics by reducing a government's exposure to self-fulfilling debt crises.

In our analysis, we consider a three-period version of the model in Lorenzoni and Werning (2019). The government follows a fiscal rule that determines stochastic primary surpluses. It finances deficits through long-term bonds and defaults whenever debt repayment is infeasible. At the beginning of each period, lenders bid on bond prices, and the government adjusts debt issuance accordingly. Importantly, we assume fiscal policy cannot response to bond price fluctuations due to political and administrative rigidities; see, for example, Conesa and Kehoe (2024).

The government's inability to adjust fiscal surplus in the short run prevents it from committing to a fixed debt level when issuing bonds, creating potential for multiple equilibria. When bond prices fall, ideally, the government would reduce deficits or limit debt issuance. However, with a fixed deficit, the only feasible response is to issue more debt, leading to higher default risk, which in turn justifies low bond prices. This paper's key contribution is to incorporate foreign reserves into the analysis.

The multiplicity in the model is dynamic. If investors expect low future borrowing, they anticipate high future bond prices. With long-term bonds, this expectation increases the bond price today, allowing the government to finance its deficit with less debt. Lower debt today reduces future debt obligations, leading to low future borrowing and validating investors' optimistic expectation. In contrast, expectations of high future borrowing lower current bond prices, forcing greater initial debt issuance, tightening future budgets, and reinforcing pessimistic expectations.

The government's only choice is the composition of its foreign portfolio. It issues debt to finance an exogenous fiscal deficit but can also issue additional debt to accumulate reserves. This financial operation provides insurance against self-fulfilling crises at intermediate deficit levels. The mechanism works as follows. The government carries higher debt and assets into the future by issuing debt to finance reserves. If lenders were to coordinate on low future bond prices, the government can cheaply repurchase its increased debt stock using reserves, relaxing its fiscal constraints in these states. This mechanism reduces future borrowing needs, undermines high borrowing expectations, and prevents self-fulfilling crises. Ultimately, holding larger reserves and debt decrease future risk, increases current bond prices, and lowers default probability.

The main result of this paper is that, in the model, the government can always eliminate multiple equilibria by increasing debt and reserves. However, this result relies on the assumption that the primary deficit is exogenous and predetermined, implying government commitment to future policies. Consequently, we abstract from dilution costs associated with long-term debt issuance.[1] This simplification highlights how debt-financed reserves provide insurance, particularly valuable when bond prices are subject to confidence-driven fluctuations.

Recent literature has explored how reserves can eliminate multiple equilibria and reduce confidence-driven fluctuations in sovereign spreads. The most closely related paper is Barbosa-Alves et al. (2024), but our paper differs in the mechanism through which reserves provide insurance. We study a Calvo-style multiple equilibria model in which the bond market remains open during crises, allowing the government to buy back bonds cheaply when investors panic. Thus, debt-financed reserves provide insurance through the state-contingent valuation of bonds. In contrast, Barbosa-Alves et al. (2024) analyzes a Cole-Kehoe-style model, where the bond market shuts down during confidence crises, rendering the state contingency of bond prices irrelevant. In their model, debt-financed reserves offer insurance by increasing liquidity. In practice, both mechanisms are likely complementary.

A key insight from these theoretical contributions is that it may be optimal for the government to accumulate reserves even when bond prices are low, especially if low prices reflect the risk of future self-fulfilling crises.[2] This finding contrasts with earlier contributions, such as Bianchi et al. (2018), where governments accumulate reserves in good times and spend them during turbulent times when spreads rise and bond prices fall.

We present new empirical evidence that during episodes of *Fiscal Stress*, defined as quarters characterized by high government bond yields, governments tend to accumulate reserves rather than deplete them to avoid rolling over high-spread debt. This observed behavior motivates the development of a theoretical framework in which reserve accumulation during periods of weak fundamentals helps prevent self-fulfilling debt crises.

Related Literature. Our paper belongs to the literature on self-fulfilling debt crises. Two canonical examples of multiplicity in sovereign debt models are Cole and Kehoe (2000) and

[1] See Hatchondo et al. (2016) for an extensive analysis of dilution costs.

[2] See Corsetti and Maeng (2023) or Hur and Kondo (2016) for related results in multiple-equilibria models.

Calvo (1988). Similar to Cole and Kehoe (2000), multiplicity in our model arises from timing assumptions. Specifically, we assume that the lenders bid on bond prices after the government commits to a fiscal surplus. For further research on Cole-Kehoe-type runs, see Bocola and Dovis (2019), Conesa and Kehoe (2017), Bianchi and Mondragon (2022) and Barbosa-Alves et al. (2024). Furthermore, Aguiar et al. (2022) examines a related form of multiplicity, in which the government issues debt before choosing whether to repay or default.

The main precursor to our paper is Lorenzoni and Werning (2019), which shows how Calvo-style multiplicity can arise when the government follows a fiscal rule. Their analysis highlights the role of debt maturity, finding that longer-term debt reduces vulnerability to multiple equilibria. We extend their model by examining how reserves can serve a similar purpose in dynamic settings with multiplicity. Our key contribution is to show that debt-financed reserves mitigate self-fulfilling crises by effectively lengthening the maturity of the government's portfolio.

Our paper also contributes to the literature on sovereign default and international reserve accumulation (see Bianchi and Lorenzoni (2022) for a recent survey). Bianchi et al. (2018) examine the role of reserves in a model with long-term debt and rollover risk driven by shifts in investor risk aversion. Bianchi and Sosa-Padilla (2023) explore the macroeconomic-stabilization role of reserves in a setting with endogenous default risk and nominal rigidities. Alfaro and Kanczuk (2009) study the joint accumulation of reserves and defaultable *one-period bonds*, finding that reserves are not optimal when debt is short-term.

Closely related to our analysis, Barbosa-Alves et al. (2024), Corsetti and Maeng (2023), and Hernández (2018) study the role of international reserves in the presence of self-fulfilling crises. Barbosa-Alves et al. (2024) and Hernández (2018) analyze a model with self-fulfilling runs, as in Cole and Kehoe (2000), and show that issuing debt to accumulate reserves is optimal only when the government's net foreign asset (NFA) position is sufficiently high. Corsetti and Maeng (2023) study joint accumulation of reserves and debt in the belief-driven sovereign risk model proposed by Aguiar et al. (2022). Their findings suggest that accumulating reserves and one-period debt is desirable because it mitigates the intra-period risk that the government may refuse to repay if the bond auction fails. In contrast to these studies, we focus on a setting with dynamic multiplicity, as in Lorenzoni and Werning (2019), and show that debt-financed reserves are optimal even when the NFA position is low.

We also contribute to the empirical literature on the evolution of debt and reserves around crisis episodes. Two important studies in this area are Gourinchas and Obstfeld (2012) and Aizenman and Sun (2012). While Gourinchas and Obstfeld (2012) focus on default episodes, our analysis centers on periods where governments continue to repay but face unusually high bond yields. Meanwhile, Aizenman and Sun (2012) examine the global financial crisis; we complement their analysis by identifying episodes where government bond yields rise above their typical levels.

Finally, our paper builds on the literature on the sustainability of debt (see Bohn (1995) and Bohn (2005)). Like this literature, we are interested in analyzing policies that promote debt sustainability. In our case, we explore the role of foreign reserves in reducing the risk of default in environments with multiple equilibria.

Outline. The paper is organized as follows. Section 2 presents the empirical motivation. Section 3 presents the model. Section 4 describes the equilibrium conditions. Section 5 studies the multiplicity in period one and the role of reserves in mitigating the risk of self-fulfilling crises. Section 6 explores the multiplicity in period zero and the implications for welfare. Section 7 concludes.

2 Empirical Motivation

In this section, we empirically test whether governments draw down reserves when bond prices are low (i.e., when yields are high). We focus on episodes of Fiscal Stress, defined as quarters where governments did not default and bond yields were at least two standard deviations above their historical mean. This definition is inspired by the financial crises literature, which characterizes Sudden Stop episodes as periods spread on government bonds is high compared to it's historical mean and where there is a sufficiently large increase in the current account.[3] We exclude default episodes from the analysis, as this paper does not address the role of reserves during such events.[4]

Our analysis uses quarterly data from 15 emerging market (EM) economies covering the $2004 - 2019$ period.[5] Our sample includes all EMs for which quarterly local-currency government bond yield data are available. We construct nine-quarter event windows centered on the date of Fiscal Stress episode. All variables, except bond yields, are expressed as a percentage of GDP and normalized to zero at time t. To ensure that increases in the ratios are not mechanically driven by falling GDP during the episodes, we fix the denominator using the level of GDP in the first quarter of the window $(t - 4)$. Appendix C provides full details about dataset and additional figures.

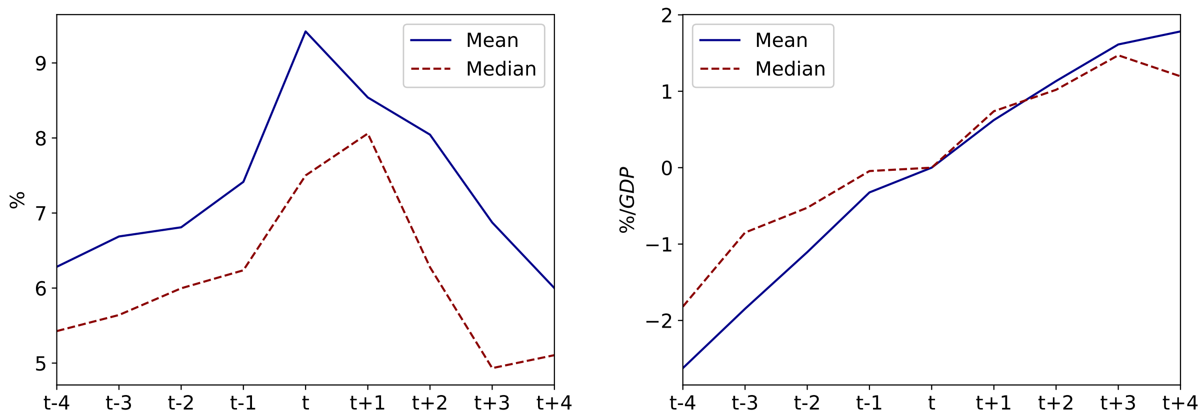
[3]See, for example, Calvo et al. (2006).

[4]See Gourinchas and Obstfeld (2012) for an analysis of NFA during default episodes.

[5]Our sample includes Brazil, Chile, China, Colombia, Indonesia, Israel, India, South Korea, Mexico, Malaysia, Peru, Philippines, Thailand, Türkiye, and South Africa.

Panel 1a illustrates the movement of bond yields around episodes of Fiscal Stress. By construction, yields spikes at time t. The nine-quarter window captures both the gradual rise that typically begins four quarters before the peak and the subsequent decline, which generally returns yields to near-average levels within four quarters afterward. Panel 1b shows that these episodes are also associated with an increase in foreign reserves—our main empirical finding. Countries tend to accumulate reserves during periods of Fiscal Stress. The median trajectory shows a brief decline in reserves at the end of the window. However, the overall cumulative change is positive, with reserves rising from 2 percentage points below their level at time t to 1 percentage point above it. This trend is even more pronounced when considering the mean.

Figure 1: Fiscal Stress Dynamics



(a) Yield

(b) Foreign Reserves

We use this empirical evidence to motivate a model in which it is optimal for the government to accumulate reserves when bond prices are low. During such periods, the confidence risk associated with multiple equilibria plays a crucial role, providing incentives for the government to accumulate reserves and reduce this risk.[6]

3 Model

We consider a small open economy that lasts for three periods, indexed by $t \in \{0, 1, 2\}$. The economy is populated by the government and a continuum of risk-neutral foreign investors

[6]See Corsetti and Maeng (2023), Barbosa-Alves et al. (2024) and Hur and Kondo (2016) for related results.

with discount factor β. The government follows an exogenous fiscal surplus rule, and default occurs whenever it cannot raise enough revenue from new debt issuance to cover its current financing needs.

The timing is as follows. At the beginning of each period, the government generates a fiscal surplus z_t, representing the total taxes collected minus the total government spending on purchases and transfers. A negative value of z_t corresponds to a primary deficit. Next, lenders bid on the price of government bonds. The variable ω_t is a sunspot that acts as an equilibrium selection device, choosing the price whenever multiple equilibria are possible. Given the price, the government issues debt or defaults at the end of the period.

3.1 Stochastic Processes

Uncertainty is modeled as follows. First, while the government runs a fixed surplus in periods zero and one, the surplus in period two z_2 is drawn from a continuous distribution $F(z_2)$ with support $[0, \overline{Z}]$.

The second source of uncertainty is confidence risk. In the event of multiplicity, we assume that a sunspot variable selects the equilibrium. With probability π, the government faces a *good sunspot* associated with a high bond price, and $\omega_t = 1$. On the other hand, with probability $1-\pi$, it encounters a *bad sunspot* linked to a low price, and $\omega_t = 0$.

The exogenous state in period zero is $s_0 = \omega_0$. In period one, it is $s_1 = \{\omega_0, \omega_1\}$, and in period two, it is $s_2 = \{\omega_0, \omega_1, z_2\}$.

3.2 Government

The government only derives utility from period two. Its preferences are given by

$$U = \mathbb{E}[u(z_2 - b_2(1-d_2)) - \gamma d_2], \tag{1}$$

where u represents the utility function, d_2 is an indicator variable that equals one if the government defaults in the final period, and γ denotes the utility cost of default. We assume that γ is sufficiently large, so the government prefers to avoid default whenever possible. This preference structure is consistent with the debt sustainability literature (see, for example, Bohn (1995)), where the government wants to finance an exogenous fiscal surplus while maintaining solvency.

In period zero, the government issues a two-period bond b_1, which pays a sequence of coupons $\kappa_1, (1-\delta)\kappa_2$ in periods one and two, respectively. In period one, the government issues a new bond i_2, which pays κ_2 in period two. The total debt obligation in period two is given by $b_2 = i_2 + (1-\delta)b_1$. We normalize parameters by setting $\kappa_1 = \frac{1}{\beta} - (1-\delta)$ and $\kappa_2 = \frac{1}{\beta}$, ensuring that the bond prices of b_1 and b_2 equal one in the absence of default risk. This structure follows the sovereign debt literature on long-term bonds (see, for example, Arellano and Ramanarayanan (2012), Hatchondo and Martinez (2009)), where $\delta = 0$ corresponds to a consol and $\delta = 1$ to a short-term bond.

The government's only choice variable is the level of international reserves in period zero. We model reserves as a one-period risk-free bond, denoted by a_1. The price of reserves is $q_a = \beta$, and they yield one unit of the endowment upon maturity. In the event of repayment, the government's budget constraints are as follows:

$$z_0 + q_0(s_0)(b_1(s_0)) = q_a a_1(s_0), \tag{2}$$

$$z_1 + q_1(s_1)(b_2(s_1) - (1-\delta)b_1(s_0)) + a_1(s_0) = \kappa_1 b_1(s_0), \tag{3}$$

$$z_2 \geq \kappa_2 b_2(s_1), \tag{4}$$

where q_0 and q_1 denote the prices of government bonds in periods zero and one, respectively. We assume that the government always honors its debt whenever feasible. That is, if there exists a level of debt b_2 that satisfies equation (3), the government repays its outstanding obligation $\kappa_1 b_1$. Similarly, in period two, the government repays $\kappa_2 b_2$ if the realized surplus z_2 is sufficient to cover the payment. We impose the following assumption:

Assumption 1. *There exists $x < \overline{Z}$ such that*

$$z_0 + \beta \left(z_1 + (1 - F(x))\, x + \beta\phi \int_0^x z_2 dF(z_2) \right) = 0.$$

Assumption 1 imposes a restriction on the expected present value of the government's surpluses, ensuring that repayment is feasible. Intuitively, if the expected sum of surpluses across periods zero, one, and two is negative, then the government lacks sufficient resources to meet its obligations, and no equilibrium without default can exist. In other words, the assumption rules out scenarios where the government's expected income is too low to sustain repayment, regardless of the realization of uncertainty.

After a default, investors receive a recovery value given by $\nu_t = \phi z_t$, where $\phi \in [0, 1]$ is the recovery rate, and the government is permanently excluded from the financial market.

3.3 International Investors

We assume that there is a continuum of risk-neutral international investors. Given the government's default policy, the non-arbitrage condition of international investors implies:

$$q_t = \beta\mathbb{E}\left[(1-d_{t+1})(\kappa_{t+1}+(1-\delta)q_{t+1}) + d_{t+1}\frac{\nu_{t+1}}{b_{t+1}}|\ s_t\right]. \tag{5}$$

3.4 Competitive Equilibrium

We are ready to define the equilibrium:

Definition 1. (Competitive Equilibrium) A competitive equilibrium consists of bond prices in the two periods, $\{q_0, q_1\}$, debt issuances in the two periods, $\{b_1, b_2\}$, reserve accumulation in the first period, $\{a_1\}$, and default rules in the two periods, $\{d_1, d_2\}$, such that:

i Given prices, debt, reserves, and default rules are consistent with the government's budget constraints (2), (3) and (4).

ii Given debt, reserves and default rules, the non-arbitrage condition of international investors (5) holds.

3.5 Discussion

Fiscal rules of this kind are standard in the literature on debt sustainability (see, for example, Bohn (1995), Bohn (2005)). One key insight from this literature is that governments cannot immediately adjust the fiscal surplus in response to fluctuations in bond prices. Instead, they often respond by adjusting debt levels, as modifying the surplus is typically costly. In our model, the government has an additional policy tool: it can also adjust the level of reserves.

For simplicity, we assume that the fiscal rule does not react to the level of government debt. As noted in Lorenzoni and Werning (2019), this assumption increases the government's vulnerability to self-fulfilling crises and helps highlight the role of reserves in preventing them. While one could allow the fiscal rule to adjust in response to bond prices, the key assumption for our analysis is that the government cannot freely adjust the surplus in the short run. This lack of flexibility opens the door to multiplicity, as the government cannot commit to a specific debt level before the bond auctions. If bond prices fall, the government would need to raise the surplus to meet its fiscal budget, but in this environment, it lacks the ability to do so.

Without loss of generality, we also assume that the government does not accumulate reserves in period one, as it would optimally choose zero. This aligns with a standard result in the literature on foreign reserves accumulation, which states that reserves are irrelevant when governments issue only short-term debt (see Alfaro and Kanczuk (2009), Bianchi et al. (2018)). In our setting, the government issues only short-term bonds in period one, which carry high interest rates due to default risk. Since the relevant state in period two is $b_2 - a_2$, the government could relax the fiscal budget in period one by selling reserves (at a higher price) to repurchase debt (at a lower price), without changing $b_2 - a_2$. Therefore, the optimal reserve choice in period one is zero.

4 Equilibrium

This section solves the model by backward induction and describes the equilibrium. We use X_t to denote the equilibrium function of x_t.

4.1 Period Two

In period two, the government begins with an outstanding debt b_2 and does not issue new bonds. Two cases are possible. If $z_2 \geq \kappa_2 b_2$, the government can repay, and current bondholders receive κ_2 per bond. If instead $z_2 < \kappa_2 b_2$, the government defaults and each bondholder receives:

$$\frac{\phi z_2}{b_2} < \kappa_2. \tag{6}$$

This implies that the government's default rule is as follows.

$$D_2(b_2, z_2) = \begin{cases} 1 & \text{if} \quad z_2 < \kappa_2 b_2, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Using the default function and the recovery value, the equilibrium bond price function in period one can be expressed as:

$$Q_1(b_2) = \beta \left[(1 - F(\kappa_2 b_2))\kappa_2 + \frac{\phi}{b_2} \int_0^{\kappa_2 b_2} z_2 dF(z_2) \right]. \tag{8}$$

Note that the price function in period one is unique.

4.2 Period One

In this section, we focus on the case where the deficits in period zero and one, z_0 and z_1, are such that the government does not default in either period. This allows us to isolate the role of confidence risk. Appendix B describes the default functions in these periods. Under repayment, the period-one budget constraint is given by:

$$z_1 + Q_1(b_2)(b_2 - (1-\delta)b_1) + a_1 = \kappa_1 b_1. \tag{9}$$

We define $\mathcal{L}_1(b_2) = Q_1(b_2)(b_2 - (1-\delta)b_1) + a_1 - \kappa_1 b_1$ as the Laffer curve of debt that includes reserves less coupon payments.

Debt Function. We define $B_2(b_1, a_1, \omega_1)$ as the equilibrium debt function. It selects the equilibrium debt from the set of candidates b_2 that satisfy the government's budget constraint. Formally we use equation (9) to define:

$$\mathbb{B}_2(b_1, a_1) = \left\{ b_2 : b_1 = \frac{z_1 + Q_1(b_2)b_2 + a_1}{\kappa_1 + (1-\delta)Q_1(b_2)} \mid b_1, a_1 \right\}. \tag{10}$$

This set contains all possible debt values that can be part of the equilibrium for any pair $\{b_1, a_1\}$. For some initial portfolios, $\mathbb{B}_2(b_1, a_1)$ contains only one value, in which case the equilibrium is unique. However, the set contains three values for some combinations of debt and reserves.

If multiplicity arises, we select an equilibrium using the sunspot variable ω_1. We denote a *good sunspot* when $\omega_1 = 1$, in which case lenders coordinate on a price consistent with the lowest value of b_2 that satisfies the equilibrium conditions. Conversely, when $\omega_1 = 0$, we refer to a *bad sunspot*, where the debt takes the highest value consistent with equilibrium conditions. We follow Ayres et al. (2018), among others, in discarding the intermediate case as a possible equilibrium. The main argument for discarding the intermediate equilibrium is that, it yields counterintuitive comparative statics, wherein higher deficits result in lower debt in equilibrium.

The debt function in period one is:

$$B_2(b_1, a_1, \omega_1) = \begin{cases} \max \mathbb{B}_2(b_1, a_1) & \text{if} \quad \omega_1 = 0, \\ \min \mathbb{B}_2(b_1, a_1) & \text{if} \quad \omega_1 = 1, \\ 0 \quad \text{otherwise.} \end{cases} \tag{11}$$

Price Function. Finally, we use the break-even condition of international investors and the debt function of the government to define the equilibrium price in period zero as:

$$Q_0(b_1, a_1) = \beta\left(\kappa_1 + (1-\delta)\mathbb{E}\left[Q_1(B_2(b_1, a_1, \omega_1))\right]\right). \tag{12}$$

As highlighted by Lorenzoni and Werning (2019), the existence of multiple equilibria in period one means that multiple price functions are consistent with equilibrium conditions in period zero. By imposing a structure on the sunspot variable, we select a price function in which lenders use the probability π of a *good sunspot* in period one to price the bond in period zero.

4.3 Period Zero

Next, we describe the equilibrium in period zero for a given state ω_0. We assume that the government begins with zero debt $b_0 = 0$ and zero reserves $a_0 = 0$.

Debt Function. Analogous to period one, we use the budget constraint in period zero to define:

$$\mathbb{B}_1(a_1) = \left\{b_1 : z_0 + Q_0(b_1, a_1)(b_1) = q_a a_1 |\ a_1\right\}. \tag{13}$$

These are the candidates' values of debt in period zero that could be part of the equilibrium for any choice of foreign reserves a_1. We define the equilibrium debt as follows:

$$B_1(a_1, \omega_0) = \begin{cases} \max \mathbb{B}_1(a_1) & \text{if} \quad \omega_0 = 0, \\ \min \mathbb{B}_1(a_1) & \text{if} \quad \omega_0 = 1, \\ 0 \quad \text{otherwise.} \end{cases} \tag{14}$$

Optimal Policy. Finally, the government's problem in period zero is:

$$V = \max_{a_1} \mathbb{E}[u(z_2 - (1-d_2)b_2) - \gamma d_2], \tag{15}$$

where $b_2 = B_2(B_1(a_1, \omega_0), a_1, \omega_1)$ and $d_2 = D_2(B_2(B_1(a_1, \omega_0), a_1, \omega_1), z_2)$. The solution to the government's problem gives decision rules for reserves A_1.

4.4 Equilibrium

We close the description of the model with the definition of a Continuation Equilibrium of the economy.

Definition 2. (Continuation Equilibrium) A Continuation equilibrium consists of a set of policies A_1, B_1, B_2, D_1, D_2, a value function for the government V and price schedules Q_0, Q_1, such that:

i Given price schedules and default and debt policy rules, A_1 solves the government problem, and V attains the maximum.

Now we will move to the analysis of multiplicity and the role of reserves. We begin by analyzing the equilibrium in period one for any combination of debt and reserves and next we move to the analyzes of the equilibrium in period zero.

5 Multiplicity in Period One

We begin our analysis by examining the equilibrium in period one. Figure 2 illustrates the equilibrium conditions. The solid line represents the Laffer curve including reserves less coupon payments, $\mathcal{L}_1(b_2)$. The dashed line represents the constant fiscal deficit $-z_1$. Equilibrium values of b_2 correspond to the points where these curves intersect. In this example, two equilibria are possible.[7]

The possibility of multiple equilibria is related to the presence of debt dilution. As b_2 increases, three forces affect the Laffer curve. First, issuing more bonds raises government income. Second, higher debt increases the default probability, lowering bond prices and reducing income. Third, falling bond prices reduce the value of outstanding debt $(1-\delta)b_1$ due to dilution.[8]

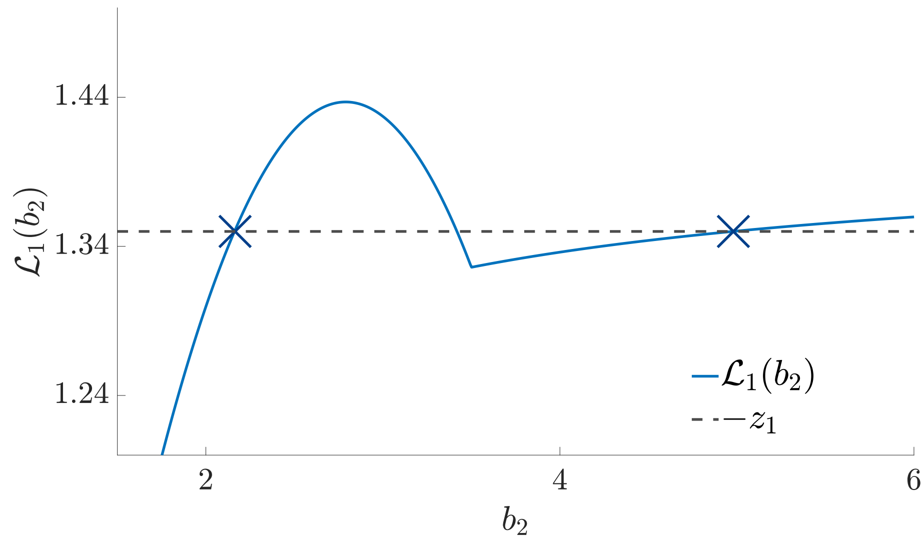
These three forces generate a Laffer curve with three distinct regions. In the first region, default risk is low and bond prices are relatively inelastic to borrowing, so the quantity effect dominates, and the curve slopes upward. As borrowing increases, the price effect becomes stronger, leading to the second region where the curve slopes downward. In the third region, default in period two becomes certain, causing bond prices to decline linearly. At these borrowing levels, new bond issuance yields constant revenue, but the dilution effect reduces

[7] We discard the intermediate equilibrium as discussed in the previous section.

[8] See Hatchondo et al. (2016) and Lorenzoni and Werning (2019) for discussions on dilution in sovereign debt models.

the value of existing debt stock $(1-\delta)b_1$. Consequently, at very high borrowing levels, revenue increases with borrowing.

Figure 2: Equilibrium Condition in Period One



Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0$, $\phi = 0.7$, $b_1 = 0.55$, $a_1 = 0$, $z_1 = -1.2$, $z_2 \sim Unif(0.55, 3.5)$.

When the government faces a Laffer curve with three regions, multiple equilibria may arise. The intuition is as follows. If investors expect the government to issue a high b_2, they anticipate a high default probability in period two and bid down bond prices. In response, the government must issue more debt to finance the deficit, validating the initial expectation. On the other hand, if investors expect a low b_2, default risk is low, bond prices remain high, and the government issues less debt, again confirming the initial expectation.

This type of multiplicity was first studied by Calvo (1988) and arises under two critical conditions: a double-peak Laffer curve and bond prices being set before the government's default decision. An alternative approach to deliver such a curve is through rare disaster risk, as shown in Ayres et al. (2018), whose model is closer to the standard Eaton-Gersovitz framework. In addition, Lorenzoni and Werning (2019) shows that imposing a lower bound on bond prices can lead to a bad equilibrium resembling a run, as in Cole and Kehoe (2000). Our focus is to understand how the presence of multiple equilibria influences the government's choices in period zero.

5.1 Debt Function

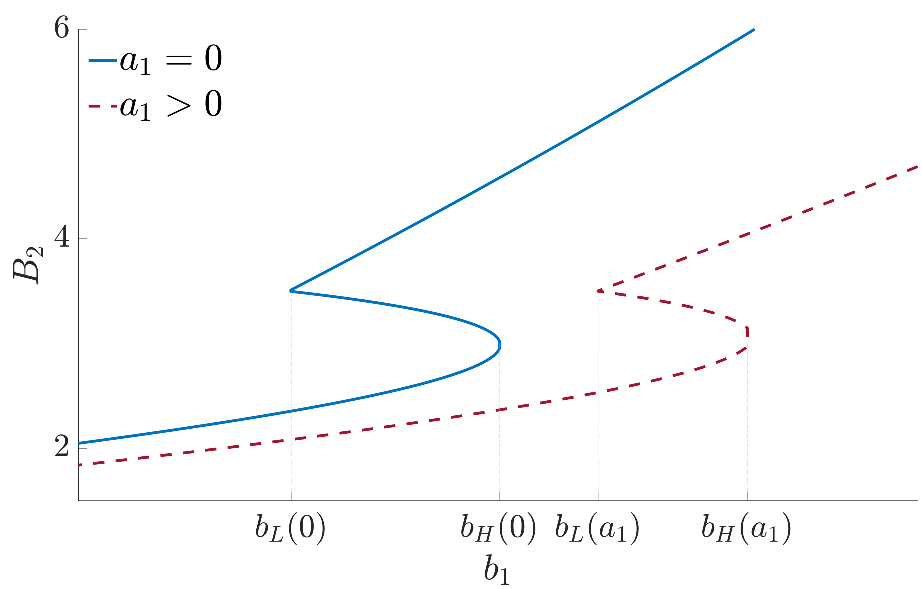
Next, we characterize the state space of debt and reserves as follows:

Definition 3. (Portfolio Regions) Let (b_1, a_1) be the initial levels of debt and reserves in period one, and let $B_2(b_1, a_1, \omega_1)$ be the debt function. The state space of (b_1, a_1) can be partitioned into the following regions:

$$\mathcal{B}_{\mathcal{L}} = \{(b_1, a_1) : B_2(b_1, a_1, \omega_1) = \mathbb{B}_2(b_1, a_1), F(B_2) < 1\}$$
$$\mathcal{B}_{\mathcal{M}} = \{(b_1, a_1) : B_2(\omega_1 = 0) = \max \mathbb{B}_2(b_1, a_1), F(B_2) = 1; B_2(\omega_1 = 1) = \min \mathbb{B}_2(b_1, a_1), F(B_2) < 1\}$$
$$\mathcal{B}_{\mathcal{H}} = \{(b_1, a_1) : B_2(b_1, a_1, \omega_1) = \mathbb{B}_2(b_1, a_1), F(B_2) = 1\}$$

This construction partitions the initial debt space b_1, conditional on a given level of reserves a_1, into three regions. In the low-debt region $\mathcal{B}_{\mathcal{L}}$, equilibrium is unique and characterized by low new borrowing and high bond prices. In the high-debt region $\mathcal{B}_{\mathcal{H}}$, the equilibrium is also unique but features high borrowing, low bond prices, and default probability of one in the final period. Finally, the multiplicity region $\mathcal{B}_{\mathcal{M}}$ allows for multiple equilibria. At these intermediate debt levels, two stable equilibria are possible: one with high borrowing and low prices and another with low borrowing and high prices.[9]

Figure 3: Borrowing and Reserves



Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0.0$, $\phi = 0.7$, $z_0 = -0.27$, $z_1 = -1.2$, $z_2 \sim Unif(0.55, 3.5)$.

Figure 3 shows a numerical example of the portfolio regions by plotting equilibrium

[9] This classification assumes repayment in period one. Formally, there exists another region where $D_1(b_1, a_1) = 1$, in which no value of b_2 satisfies the government's budget constraint.

borrowing as a function of initial debt b_1 for two reserve levels a_1. The solid line represents an economy without reserves, while the dashed line corresponds to an economy with positive reserves. The functions $b_L(a_1)$ and $b_H(a_1)$ denote the boundaries of the multiplicity region as a function of reserves. To analyze how reserves influence confidence risk, we characterize these boundaries in two steps, beginning with the following lemma that establishes conditions for their existence.

Lemma 1. *(Thresholds of debt) Suppose Assumption 1 holds. Then, for a given level of reserves a_1, there exist two thresholds of debt in period one, $b_H(a_1), b_L(a_1)$ such that; $b_1 \in \mathcal{B}_{\mathcal{M}}$ if and only if $b_H(a_1) \geq b_1 \geq b_L(a_1)$*

Proof. See Appendix A.1. □

Lemma 1 establishes the existence of debt thresholds at which the government faces the risk of multiple equilibria. For a given level of reserves, the equilibrium is unique and default risk is low when debt is below the lower threshold, while levels above the upper threshold are associated with a default probability of one in the last period. We now examine how these thresholds vary with the level of reserves.

Lemma 2. *(Thresholds of Debt and Reserve) Suppose $\hat{a}_1 > \tilde{a}_1 \geq 0$. Then:*

1. $b_L(\hat{a}_1) > b_L(\tilde{a}_1)$
2. $b_H(\hat{a}_1) > b_H(\tilde{a}_1)$.

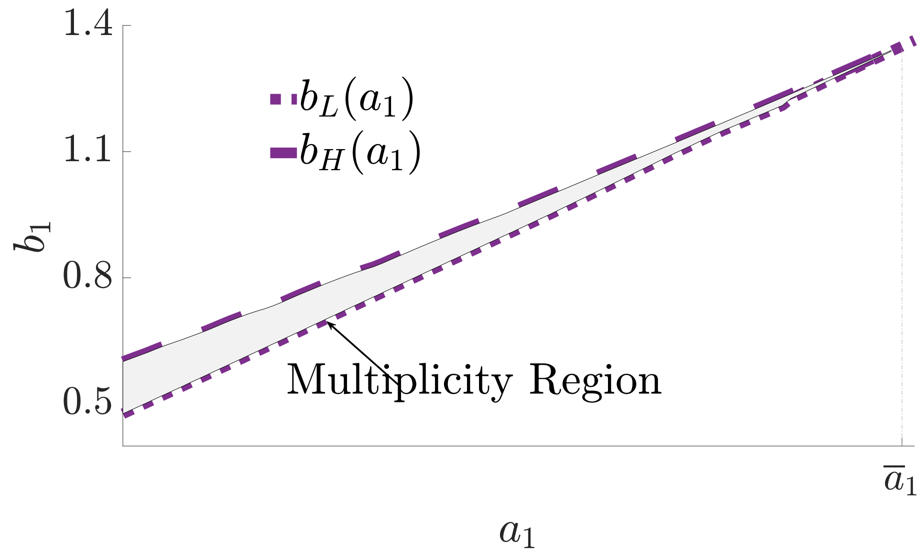
Proof. See Appendix A.2. □

Lemma 2 compares the boundaries of the multiplicity regions across different levels of reserves and shows that both $b_L(a_1)$ and $b_H(a_1)$ are increasing functions of a_1. Figure 4 plots these boundaries as functions of reserves. A key insight from the figure is the existence of a threshold level of reserves, denoted $\overline{a}_1$, such that for any $a_1 > \overline{a}_1$, the boundaries coincide, $b_L(a_1) = b_H(a_1)$. At this point, the government fully eliminates the possibility of multiple equilibria in period one. We formalize this result in the following proposition.

Proposition 1. *Suppose Assumption 1 holds. There exists a level of reserves $\overline{a}_1$ such that $b_L(\overline{a}_1) = b_H(\overline{a}_1)$ and the equilibrium in period one is unique for all $a_1 \geq \overline{a}_1$.*

Proof. See Appendix A.3. □

Figure 4: Boundaries of the Multiplicity Region



Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0.0$, $\phi = 0.7$, $z_0 = -0.27$, $z_1 = -1.2$, $z_2 \sim Unif(0.55, 3.5)$.

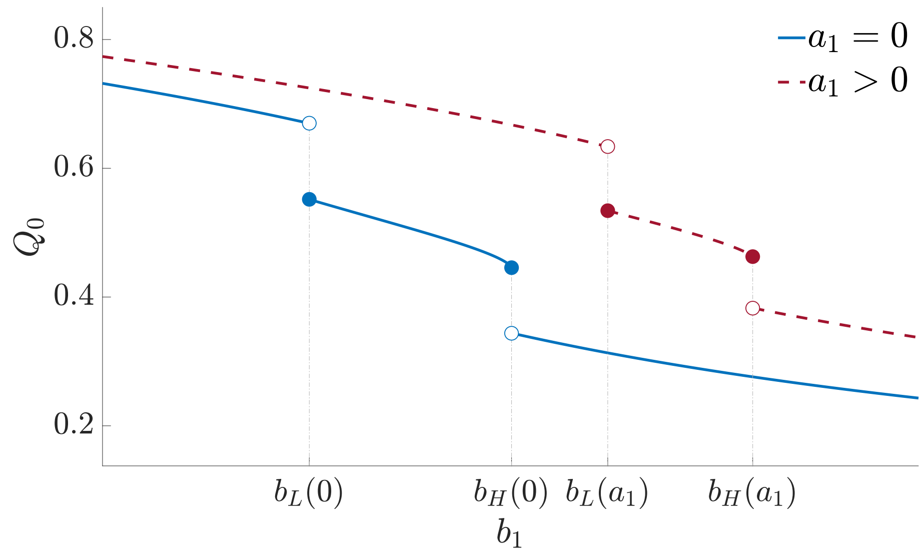
If the government chooses $\overline{a}_1$, it effectively eliminates the possibility of multiple equilibria, regardless of the level of b_1 required to finance these reserves. The intuition is as follows. At $\overline{a}_1$, the government issues a high level of debt in the first period to accumulate a significant amount of reserves. When initial debt is sufficiently high, the dilution effect dominates the price effect. That is, additional borrowing reduces the value of outstanding debt more than it decreases the revenue from new issuance. As a result, the Laffer curve becomes strictly increasing in new borrowing, eliminating the possibility for multiple equilibria, since higher borrowing always leads to higher income.

The key implication of this result is that the government can eliminate confidence-driven risk by accumulating reserves, as long as repayment remains feasible. However, since holding reserves is costly in practice, we next analyze the effect of reserve accumulation on bond prices in period zero.

5.2 Price Function

We now analyze how multiplicity affects period-zero bond price function. In Figure 5, we substitute the equilibrium debt function $B_2(b_1, a_1, \omega_1)$ into equation (12) to express the bond price as a function of debt and reserves in period zero. The solid blue lines represent the price function without reserves, while the red dashed lines show the case with positive reserves. Two jumps occur at the boundaries of the multiplicity region, reflecting changes in future borrowing outcomes due to multiplicity in period one.

Figure 5: Period-Zero Price Function



Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0.0$, $\phi = 0.7$, $z_0 = -0.27$, $z_1 = -1.2$, $z_2 \sim Unif(0.55, 3.5)$.

When b_1 is just below $b_L(a_1)$, future borrowing is unique and low, expected dilution is minimal, and the bond price is high. However, a small increase in b_1, holding a_1 fixed, moves the economy into the multiplicity region. There, two equilibria exist: one with low future borrowing and another with high. The price of the bond reflects the weighted average of these outcomes, and the possibility of high borrowing lowers the expected future value of the bond, causing a drop in the current price.

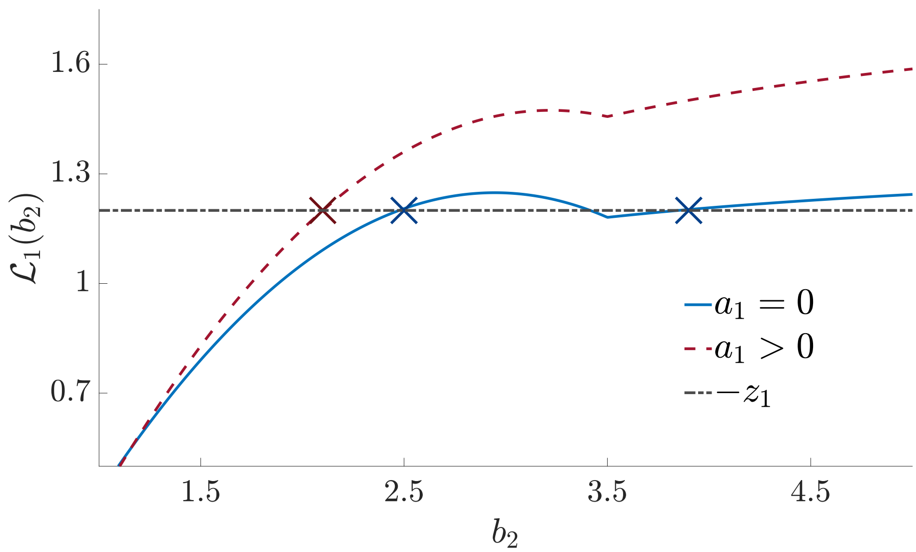
A similar discontinuity arises at the upper boundary $b_H(a_1)$. Increasing b_1 slightly beyond this threshold eliminates the *good equilibrium*, leaving only a high-borrowing outcome with high dilution. As a result, the period-zero price drops as a result of the anticipated deterioration in future bond values.

5.3 Debt-Financed Reserves

Our previous analysis characterized how the uniqueness of period one equilibrium depends on the combination of debt and reserves. However, in period zero, the government faces a trade-off as accumulating reserves requires issuing more debt. In this subsection, we analyze whether reserves can be self-financing once their positive effect on bond prices is considered. To explore this, we study a financial operation in which the government issues debt in period zero to accumulate reserves.

Figure 6 compares the equilibrium of period one in two economies with the same market value of their government portfolios at period-zero prices but different levels of reserve accumulation. The solid blue line represents the Laffer curve when the government issues only enough debt to finance its period-zero deficit, while the red dashed line represents the case where additional debt is issued to accumulate reserves. In this example, the economy with reserves exhibits a unique equilibrium in period one, whereas the one without faces multiple equilibria.

Figure 6: Equilibrium Condition in Period One



Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0.0$, $\phi = 0.7$, $z_0 = -0.27$, $z_1 = -1.2$, $z_2 \sim U(0.55, 3.5)$.

The intuition is that reserves provide additional resources in period one, regardless of bond prices. If lenders coordinate on low bond prices, the market value of the debt used to finance reserves also falls. Thus, by issuing more debt in period zero to accumulate reserves, the government can relax its budget constraint precisely in states where bond prices are low, mitigating the risk of multiplicity. Effectively, under the *bad equilibrium*, the government generates more revenue than needed to finance the deficit, making an equilibrium with low bond prices unsustainable when the government holds both higher debt and higher reserves.

To further understand the mechanisms at play, we decompose the effect of reserve accumulation into two components. First, by eliminating multiplicity in period one, reserves raise bond prices in period zero, reducing the required borrowing b_1 to finance a given deficit z_0. This in turn lower b_2. Second, reserve accumulation steepens the Laffer curve by increasing initial debt, which amplifies the dilution effect of additional borrowing. Under the bad equilibrium with low bond prices, this stronger dilution effect relaxes the government's budget constraint. As illustrated in Figure 6, the steeper Laffer curve renders bad equilibrium unsustainable, ruling out multiplicity. We formalize this argument in the following lemma.

Lemma 3. *(Elasticity of the Debt Laffer Curve) Consider two economies with portfolios* $\{\hat{b}_1, \hat{a}_1\}$ *and* $\{\tilde{b}_1, \tilde{a}_1\}$ *such that both have the same market value at period-zero prices but* $\hat{a}_1 > \tilde{a}_1$. *Let* $\hat{\mathcal{L}}_1, \tilde{\mathcal{L}}_1$ *denote the corresponding Laffer curves. Then:*

$$\frac{\partial \hat{\mathcal{L}}_1 / \partial b_2}{\hat{\mathcal{L}}_1} b_2 > \frac{\partial \tilde{\mathcal{L}} / \partial b_2}{\tilde{\mathcal{L}}} b_2 \quad \textit{if} \quad \partial \hat{\mathcal{L}} / \partial b_2 > 0$$
$$\frac{\partial \hat{\mathcal{L}} / \partial b_2}{\hat{\mathcal{L}}} b_2 < \frac{\partial \tilde{\mathcal{L}} / \partial b_2}{\tilde{\mathcal{L}}} b_2 \quad \textit{if} \quad \partial \hat{\mathcal{L}} / \partial b_2 < 0$$

Proof. See Appendix A.4. □

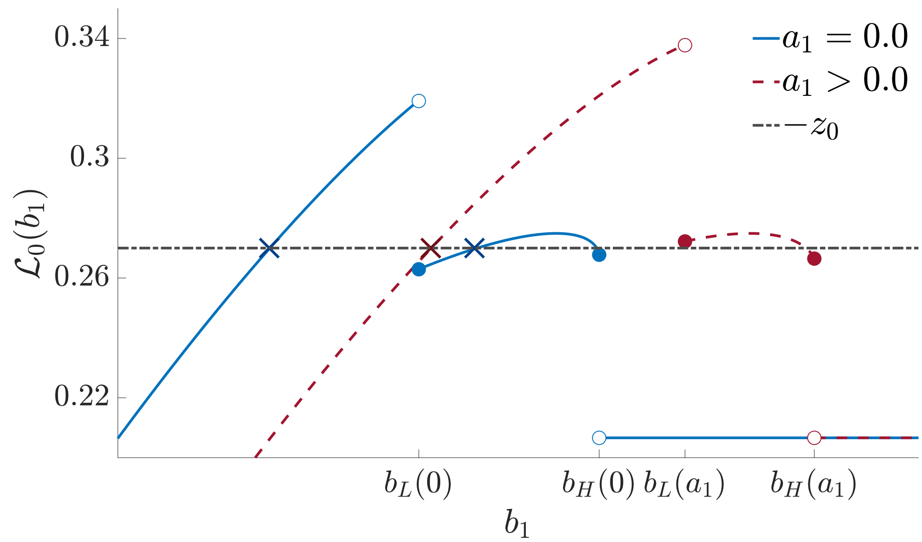
6 Multiplicity in Period Zero

We now analyze the equilibrium in period zero to fully capture the interaction between confidence risk and reserves. Multiplicity in period one affects period zero through its impact on the price function. Figure 7 illustrates this relationship by plotting period-zero equilibrium condition. The Laffer curve minus reserves in period zero exhibits two drops in the boundaries of the multiplicity region reflecting price jumps discussed in Figure 5. Consequently, for some levels of z_0, multiple equilibria emerge.

We consider the scenario in which the government, shown by the solid blue line, does not accumulate reserves. Without reserves, a sunspot in period zero may lead to a good equilibrium with low debt and high bond prices. In this equilibrium, b_1 is in the safe zone, eliminating uncertainty in period one and reducing future borrowing. Lower future borrowing reinforces high bond prices, resulting in an equilibrium with low initial debt and favorable financing condition.

In contrast, the second equilibrium features low bond prices in period zero, leading to higher initial debt. This raises uncertainty about borrowing in period one, as future coordination on low prices could force the government to borrow even more. This expectation justifies the low initial prices. Thus, the equilibrium is characterized by high debt, uncertainty about future borrowing, and low bond prices.

Figure 7: Equilibrium Condition in Period Zero



Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0.0$, $\phi = 0.7$, $z_0 = -0.27$, $z_1 = -1.2$, $z_2 \sim U(0.55, 3.5)$.

However, the government can issue more debt b_1 to accumulate reserves a_1, illustrated by the red dashed line. In this case, the equilibrium is unique. The higher stock of long-term bonds provides insurance against future crises, preventing multiplicity in period one. By financing the period-zero deficit with a portfolio that ensures uniqueness in period one, the government secures high bond prices and avoids confidence-driven risk.

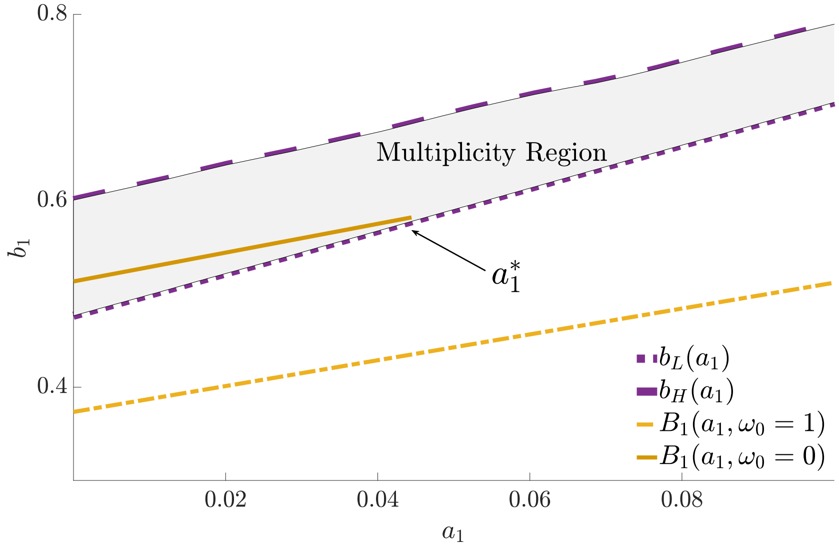
6.1 Self-Fulfilling Crises and Reserves

This section analyzes the government's vulnerability to self-fulfilling crises. Figure 8 illustrates how the equilibrium outcomes depend on the government's portfolio. As in Figure 4, we plot the boundaries of the multiplicity region in purple, along with the combinations of debt and reserves consistent with the period-zero budget constraint.

Figure 8 shows an economy with an intermediate initial deficit z_0, where fundamentals are weak. Although the government can avoid default in period zero, it remains vulnerable to a self-fulfilling crisis that increases the default probability in the final period. The solid yellow line represents combinations of debt and reserves that finance the period-zero deficit under the *bad equilibrium*, where low reserve accumulation pushes the required debt level into the multiplicity region of period one. In contrast, the dashed yellow line represents the equilibrium condition when the sunspot in period zero selects the *good equilibrium*. In this case, for all levels of reserve accumulation, the government can finance the deficit with a portfolio that keeps the period-one equilibrium outside the multiplicity region. As a result, the equilibrium in period one is unique, with both current debt issuance and future borrowing remaining low.

With zero reserves and under the *bad sunspot*, the initial debt level is high, and the government faces multiple possible levels of future borrowing. However, by increasing debt and accumulating reserves, the government moves along the solid yellow line and eventually reaches the boundary of the multiplicity region at a_1^*, where multiplicity is eliminated. At this point, the improvement in bond prices from eliminating multiplicity shifts the government onto the dashed yellow line, enabling it to finance the period-zero deficit with low debt and low future borrowing.

Figure 8: Portfolio Composition and Multiplicity

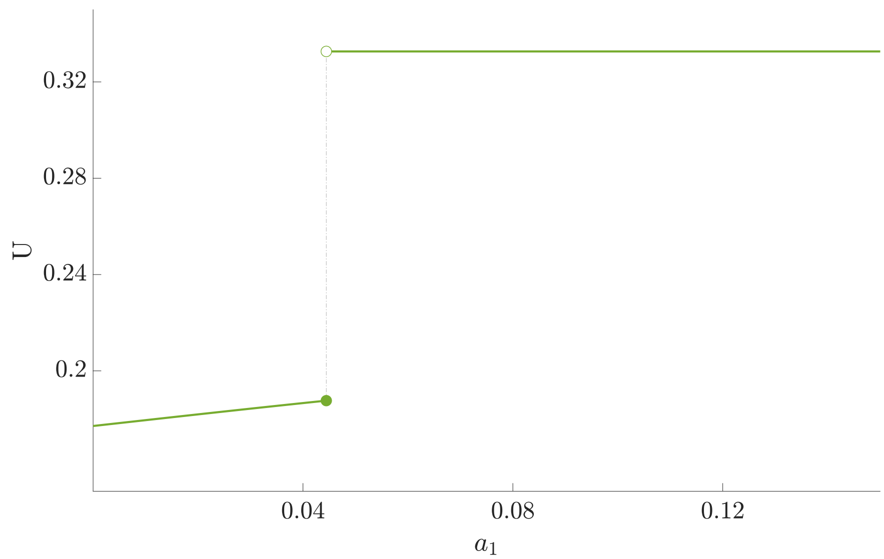


Although a formal proof that $\overline{a}_1 > a_1^*$ is unavailable, this relationship holds in all our numerical examples. The key insight is that the government does not need to accumulate reserves until the boundaries of the multiplicity region coincide to eliminate multiplicity. As reserve accumulation raises bond prices, the value of the government's portfolio in period zero increases, allowing it to prevent multiplicity with a lower level of reserves.

6.2 Optimal Reserves

We now assess the welfare implications of reserves and the government's optimal reserve holdings. Figure 9 illustrates welfare as a function of reserves. When reserves are zero, multiple equilibria may arise. As reserves increase, debt and future borrowing during crises decrease, raising welfare. With linear utility, welfare increases proportionally with reserves. Once reserves reach the threshold that ensures a unique equilibrium, welfare jumps discretely. Beyond this point, welfare remains constant, as he relative price of reserves and debt in period zero keeps period-one borrowing b_2 unchanged, making the government indifferent across reserve levels that eliminate multiplicity.

Figure 9: Welfare



Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0.0$, $\phi = 0.7$, $z_0 = -0.27$, $z_1 = -1.2$, $z_2 \sim Unif(0.55, 3.5)$.

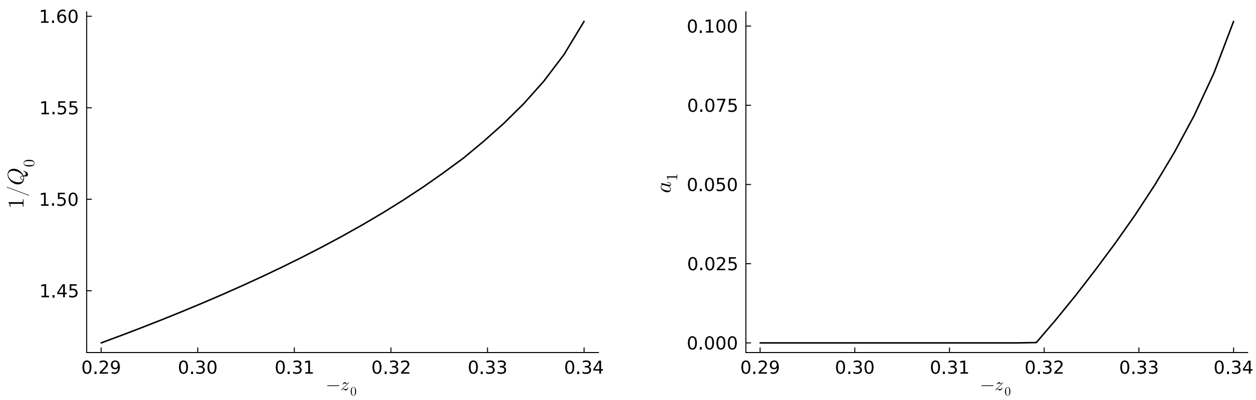
6.3 Illustration of Reserves Accumulation During Crises

We conclude this section by showing how the model's mechanisms can account for the empirical patterns identified earlier. This proof-of-concept illustrates how the model explains why a government might find it optimal to accumulate reserves during periods of high sovereign bond yields.

Figure 10 presents a comparative statics exercise in which we vary the initial government deficit z_0, holding all other parameters fixed as in Section 4. We plot the equilibrium levels of debt and reserves in the first period. Since the government is indifferent among reserve levels that ensure uniqueness, we assume it chooses the minimum level consistent with a unique equilibrium.

In this exercise, a higher initial deficit increases risk in the first period. The government responds by issuing more debt, which, in turn, raises the equilibrium bond yield due to increased default risk, as illustrated in Panel 10a. Crucially, as the need to finance larger deficits raises risk, the level of reserves required to eliminate self-fulfilling crises also increases. As a result, the government accumulates more reserves. In the model, when bond prices exhibit confidence-driven fluctuations, the government finds it optimal to accumulate reserves even when yields are high.

Figure 10: Fiscal Stress in the Model



(a) Yield (b) Foreign Reserves

Note: This figure is computed assuming $\beta = 0.95$, $\delta = 0$, $\phi = 0.7$, $z_1 = -1.2$, $z_2 \sim Unif(0.55, 3.5)$.

Discussion. The analysis isolates a clean mechanism through which reserves can mitigate confidence-driven fluctuations in sovereign bond prices when markets remain open, and we use numerical examples to illustrate how reserve accumulation can collapse the low-price equilibrium by weakening the feedback-loop between bond priced and default risk. A natural next step is a quantitative exercise that discipline rollover and confidence risk. A quantitative framework would also allow the reserve motive studied here to be confronted with other prominent motives—most notably exchange-rate management and financial-stability objectives of central banks—and to evaluate how these motives interact with the sovereign's balance-sheet and rollover considerations.

7 Conclusions

This paper presents a model in which self-fulfilling crises arise from investor expectations of high future borrowing. A loss of confidence lowers current bond prices, forcing the government to issue more debt, which validates the initial belief. We extend the literature by introducing foreign reserves as risk-free assets that the government can accumulate to strengthen its financial position.

We establish that debt-financed reserves can restore uniqueness by insuring against future crises, improving current bond prices, and eliminating multiple equilibria. Crucially, this strategy is optimal even when economic fundamentals are weak. A promising direction for future research is to extend the model for quantitatively analysis of reserves during such episodes.

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A Proofs

A.1 Proof of Lemma 1

Proof. Recall that the government's budget constraint in period one is

$$z_1 + Q_1(b_2)(b_2 - (1-\delta)b_1) + a_1 = \kappa_1 b_1. \tag{16}$$

Taking total derivative with respect to b_1, we obtain

$$\begin{aligned} 0 + \frac{\partial Q_1(b_2)}{\partial b_2}\frac{\partial b_2}{\partial b_1}\left[b_2 - (1-\delta)b_1\right] + Q_1(b_2)\left[\frac{\partial b_2}{\partial b_1} - (1-\delta)\right] + 0 = \kappa_1, \\ \frac{\partial b_2}{\partial b_1}\left[\frac{\partial Q_1(b_2)}{\partial b_2}(b_2 - (1-\delta)b_1) + Q_1(b_2)\right] = \kappa_1 + (1-\delta)Q_1(b_2). \end{aligned} \tag{17}$$

Rearranging terms:

$$\frac{\partial b_2}{\partial b_1} = \frac{\left[\kappa_1 + (1-\delta)Q_1(b_2)\right]}{\left[\frac{\partial Q_1(b_2)}{\partial b_2}(b_2 - (1-\delta)b_1) + Q_1(b_2)\right]} \tag{18}$$

$$= \frac{\left[\frac{z_1 + a_1 + Q_1(b_2)b_2}{b_1}\right]}{\left[\frac{\partial Q_1(b_2)}{\partial b_2}(b_2 - (1-\delta)b_1) + Q_1(b_2)\right]}. \tag{19}$$

For all $b_2 \geq \overline{Z}$, we use the closed-form expression for Q_1:

$$Q_1(b_2) = \left[\frac{\beta\phi}{b_2}\int_0^{\overline{Z}} z_2 dF(z_2)\right] = \frac{\beta\phi\mathbb{E}[z_2]}{b_2}, \tag{20}$$

$$Q_1(b_2)b_2 = \beta\phi\mathbb{E}[z_2], \tag{21}$$

$$\frac{\partial Q_1(b_2)}{\partial b_2} = -\frac{\beta\phi\mathbb{E}[z_2]}{b_2^2}. \tag{22}$$

Define $b_1 = b_L$ such that $B_2 = \overline{Z}$. That is,

$$b_L(a_1) = \frac{z_1 + Q_1(\overline{Z})\overline{Z} + a_1}{\kappa_1 + (1-\delta)Q_1(\overline{Z})}. \tag{23}$$

Given $b_2 \geq \overline{Z}$, we simplify the derivative:

$$\frac{\partial b_2}{\partial b_1} = \frac{\left[\frac{z_1 + a_1 + \beta\phi\mathbb{E}[z_2]}{b_1}\right]}{-\frac{\beta\phi\mathbb{E}[z_2]b_2}{b_2^2} - \frac{\partial Q_1(b_2)}{\partial b_2}(1-\delta)b_1 + \frac{\beta\phi\mathbb{E}[z_2]}{b_2}} = -\frac{\left[\frac{z_1 + a_1 + \beta\phi\mathbb{E}[z_2]}{b_1}\right]}{\frac{\partial Q_1(b_2)}{\partial b_2}(1-\delta)b_1} > 0. \tag{24}$$

Thus, for all $b_1 \geq b_L$, B_2 is increasing in b_1.

Next, define $b_1 = b_H$ such that for all $0 \leq b_1 \leq b_H$, we have

$$\left[\frac{\partial Q_1(b_2)}{\partial b_2}(b_2 - (1-\delta)b_1) + Q_1(b_2)\right] > 0. \tag{25}$$

This means that B_2 is increasing in b_1 in two regions:

- For $b_1 \geq b_L$, where $B_2 \geq \overline{Z}$ and the government defaults in period two with probability one.
- For $0 \leq b_1 \leq b_H$, where $B_2 < \overline{Z}$ and repayment is feasible.

It follows that $b_1 \in \mathcal{B}_\mathcal{M}$ if and only if $b_H(a_1) \geq b_1 > b_L(a_1)$.

□

A.2 Proof of Lemma 2

Proof. Recall that for any a_1, $b_L(a_1)$ is associated with $B_2 = \overline{Z}$. Using budget constraint in period one, we can write

$$b_L(a_1) = \frac{z_1 + Q_1(\overline{Z})\overline{Z} + a_1}{\kappa_1 + (1-\delta)Q_1(\overline{Z})}. \tag{26}$$

Taking the derivative with respect to a_1, we obtain:

$$\frac{\partial b_L(a_1)}{\partial a_1} = \frac{1}{\kappa_1 + (1-\delta)Q_1(\overline{Z})} > 0. \tag{27}$$

Next, define the Laffer curve as:

$$h(b_1, a_1) = \max_{\{b_2 | F(b_2) < 1\}} Q_1(b_2)(b_2 - (1-\delta)b_1) + a_1 - \kappa_1 b_1 \tag{28}$$

and let $\bar{b}_2(b_1, a_1)$ denote the maximizer:

$$\bar{b}_2(b_1, a_1) = \operatorname*{argmax}_{\{b_2 | F(b_2) < 1\}} Q_1(b_2)(b_2 - (1-\delta)b_1) + a_1 - \kappa_1 b_1. \tag{29}$$

Then define $b_H(a_1)$ implicitly as the value of b_1 satisfying

$$b_H(a_1) = \frac{z_1 + Q_1(\bar{b}_2)\bar{b}_2 + a_1}{\kappa_1 + (1-\delta)Q_1(\bar{b}_2)}, \tag{30}$$

which can be rearranged as:

$$z_1 + Q_1(\bar{b}_2)(\bar{b}_2 - (1-\delta)b_H) + a_1 = \kappa_1 b_H. \tag{31}$$

Taking total derivative with respect to a_1:

$$\frac{\partial Q_1}{\partial b_2}\left[\frac{\partial \bar{b}_2}{\partial b_1}\frac{\partial b_H}{\partial a_1}+\frac{\partial \bar{b}_2}{\partial a_1}\right]\left[\bar{b}_2-(1-\delta)b_H\right]+Q_1(\bar{b}_2)\left[\frac{\partial \bar{b}_2}{\partial b_1}\frac{\partial b_H}{\partial a_1}+\frac{\partial \bar{b}_2}{\partial a_1}-(1-\delta)\frac{\partial b_H}{\partial a_1}\right]+1=\kappa_1\frac{\partial b_H}{\partial a_1}. \tag{32}$$

Rearranging terms:

$$\begin{aligned}\frac{\partial \bar{b}_2}{\partial a_1}\left[\frac{\partial Q_1}{\partial b_2}\left(\bar{b}_2-(1-\delta)b_H\right)+Q_1(\bar{b}_2)\right]+\frac{\partial b_H}{\partial a_1}\frac{\partial \bar{b}_2}{\partial a_1}\left[\frac{\partial Q_1}{\partial b_2}\left(\bar{b}_2-(1-\delta)b_H\right)+Q_1(\bar{b}_2)\right]+1 \\ =\frac{\partial b_H}{\partial a_1}\left[\kappa_1+Q_1(\bar{b}_2)(1-\delta)\right].\end{aligned} \tag{33}$$

Using the definition of $\bar{b}_2$, we know that $\left[\frac{\partial Q_1}{\partial b_2}\left(\bar{b}_2-(1-\delta)b_H\right)+Q_1(\bar{b}_2)\right]=0$:

$$\frac{\partial b_H}{\partial a_1}\left[\kappa_1+Q_1(\bar{b}_2)(1-\delta)\right]=1, \tag{34}$$

$$\frac{\partial b_H}{\partial a_1}=\frac{1}{\kappa_1+Q_1(\bar{b}_2)(1-\delta)} \tag{35}$$

$$=\frac{1}{\frac{1}{\beta}+(1-\delta)(Q_1(\bar{b}_2)-1)} \tag{36}$$

$$>0. \tag{37}$$

Therefore, $b_L(\hat{a}_1)>b_L(\tilde{a}_1)$ and $b_H(\hat{a}_1)>b_H(\tilde{a}_1)$ for all $\hat{a}_1>\tilde{a}_1\geq 0$. □

A.3 Proof of Proposition 1

Proof. Suppose $\bar{b}_2(b_H(\bar{a}_1),\bar{a}_1)=\overline{Z}$. Then, from the expressions of $b_L(a_1)$ and $b_H(a_1)$, it follows that $b_L(\bar{a}_1)=b_H(\bar{a}_1)$. Also, by the definition of $\bar{b}_2$, we have

$$\frac{\partial Q_1}{\partial b_2}(b_2-(1-\delta)b_1)+Q_1(b_2)=0, \tag{38}$$

$$\frac{\frac{\partial Q_1}{\partial b_2}b_2+Q_1(b_2)}{\frac{\partial Q_1}{\partial b_2}(1-\delta)}=b_1. \tag{39}$$

Evaluating at $b_2 = \overline{Z}$:

$$\frac{\frac{\partial Q_1}{\partial b_2}\Big|_{b_2=\overline{Z}}\overline{Z} + Q_1(\overline{Z})}{\frac{\partial Q_1}{\partial b_2}\Big|_{b_2=\overline{Z}}(1-\delta)} = \frac{z_1 + Q_1(\overline{Z})\overline{Z} + \overline{a}_1}{\kappa_1 + (1-\delta)Q_1(\overline{Z})}, \tag{40}$$

$$\overline{a}_1 = \frac{\frac{\partial Q_1}{\partial b_2}\Big|_{b_2=\overline{Z}}\overline{Z} + Q_1(\overline{Z})}{\frac{\partial Q_1}{\partial b_2}\Big|_{b_2=\overline{Z}}(1-\delta)} \left[\kappa_1 + (1-\delta)Q_1(\overline{Z})\right] - z_1 - Q_1(\overline{Z})\overline{Z}. \tag{41}$$

□

A.4 Proof of Lemma 3

Proof. For any b_1, we can write

$$\frac{\partial \mathcal{L}(b_1, b_2)}{\partial b_2} = Q_1(b_2) + b_2\frac{dQ(b_2)}{db_2} - (1-\delta)b_1\frac{dQ_1(b_2)}{db_2}. \tag{42}$$

Also, note that $\frac{dQ_1(b_2)}{db_2} < 0$ and b_1 only enters the third term in the above equation. Therefore, if $\hat{b}_1 > \tilde{b}_1$, then

$$\frac{\partial \mathcal{L}(\hat{b}_1, b_2)}{\partial b_2} > \frac{\partial \mathcal{L}(\tilde{b}_1, b_2)}{\partial b_2}. \tag{43}$$

Lastly, we also know that $\mathcal{L}(\hat{b}_1, b_2) < \mathcal{L}(\tilde{b}_1, b_2)$ for all b_2. Then, the lemma follows. □

B Default Functions in Period Zero and One

B.1 Period Zero

In period zero, the budget constraint is given by:

$$z_0 + Q_0(b_1, a_1)(b_1) = q_a a_1. \tag{44}$$

The government defaults if its fiscal surplus and borrowing capacity fall short of its obligations. Formally, the default function is

$$D_0 = \begin{cases} 1 & \text{if} \quad z_0 + m_0 < 0, \\ 0 & \text{otherwise,} \end{cases} \tag{45}$$

where m_0 denotes the peak of the Laffer curve defined as:

$$m_0 = \max_{b_1, a_1} \left\{ Q_0(b_1, a_1) b_1 - q_a a_1 \right\}. \tag{46}$$

The peak m_0 is not affected by the realization of the sunspot in period zero but does depend on the probability of a bad sunspot in period one because of its effect on the bond price function Q_0.

B.2 Period One

We define the peak of the Laffer curve, including reserves, as a function of the initial portfolio:

$$m_1(b_1, a_1) = \max_{b_2} \left\{ Q_1(b_2)(b_2 - (1-\delta)b_1) + a_1 \right\}. \tag{47}$$

The government repays in period one if:

$$z_1 + m_1(b_1, a_1) \geq \kappa_1 b_1. \tag{48}$$

Accordingly, the default function in period one is given by:

$$D_1(b_1, a_1) = \begin{cases} 1 & \text{if} \quad z_1 + m_1(b_1, a_1) < \kappa_1 b_1, \\ 0 & \text{otherwise.} \end{cases} \tag{49}$$

Both the peak of the Laffer curve and the default function are independent of the sunspot realization. When $D_1(b_1, a_1) = 1$, the government defaults and is excluded from the financial markets, so $B_2(b_1, a_1, \omega_1) = 0$.

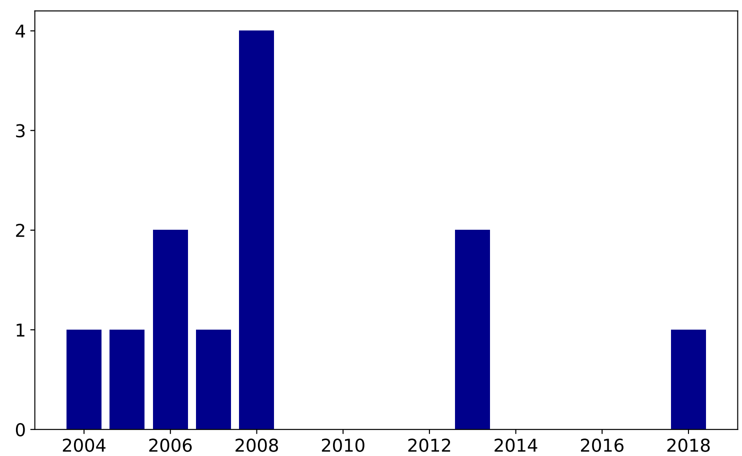
C Empirical Appendix

C.1 Data Sources

- **Spreads:** We focus on 3-month local-currency bond yields and the source is Bloomberg.
- **International Reserves:** We focuse on "Total Reserves excluding Gold". The source is IMF's International Financial Statistics.
- **Debt:** We focus on "Total General Government Gross Debt". The source is IMF's Sovereign Debt Investor Base assembled by Serkan Arslanalp and Takahiro Tsuda.
- **GDP:** We focus on "Nominal, US Dollars". The source is IMF's International Financial Statistics.

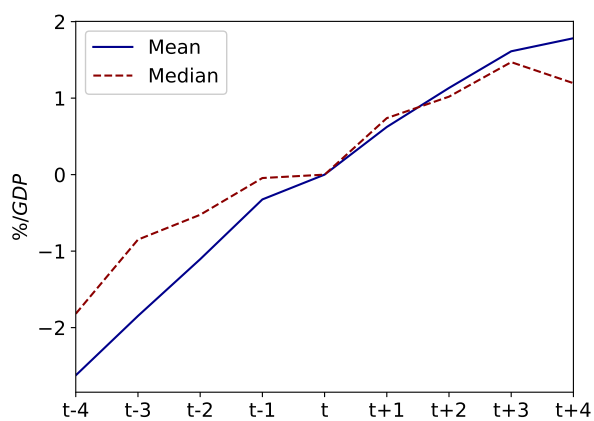
C.2 Additional Plots

Figure 11: Number of Episodes



Source: Bloomberg.

(a) Net Foreign Assets



(b) Total Debt

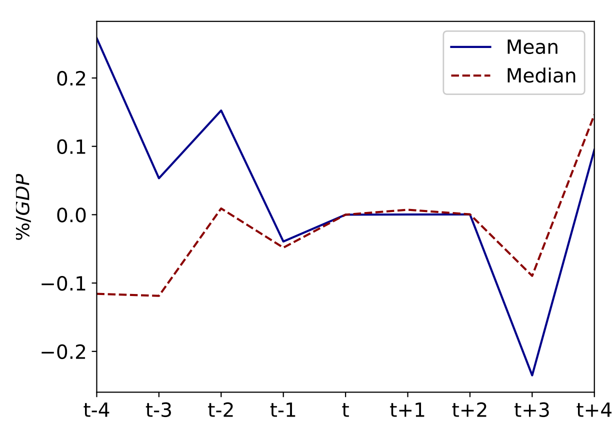


Figure 12: Fiscal Stress Dynamics